Economic Theory as a Guide for the Specification and Interpretation of Empirical Health Production Functions

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Abstract

In this paper we employ a static model of utility maximization with health production to derive precise interpretations of estimated effects of observable inputs on health outcomes. We argue that if omitted inputs are not properly controlled for, then estimated marginal products of health inputs cannot be easily interpreted. Using a general theoretical model, we propose empirical specifications to control for omitted inputs. The resulting "effects" one can estimate using such specifications do not correspond exactly to the marginal products of the observed inputs on health. Using the theoretical model, however, we establish some bounds on the "true" marginal products

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of the observed inputs when one uses empirical specifications compatible with economic theory. In particular, we show that when some key health inputs are omitted from the regression then the estimated coefficient on an observed, health improving input will likely to be negatively biased. We show some preliminary empirical evidence to support our methodology using Behavioral Risk Factor Surveillance System (BRFSS) data.

1 Introduction.

To make informed recommendations, health policy analysts need to understand how inputs to health production functions affect measurable health outcomes. Two key issues make this a difficult task. First, individuals' choices of health inputs likely depend on unobserved to the researcher baseline health characteristics and individuals' unobserved abilities to make use of the inputs. As a consequence, consumers' choices of the levels of health inputs are likely to be statistically endogenous determinants for the estimation of health production functions. Researchers have long recognized how the failure to control for the endogeneity of the health inputs can lead to biased and inconsistent estimates of the marginal effects of health inputs on health outcomes. They have used a variety of approaches to address these issues, such as better measures of health and individual productivity, experimentally assigned health inputs, and instrumental variables, natural experiments, and regression discontinuity models.

The second issue arises because one almost never can observe all of the inputs chosen by individuals to affect their health outcomes. Suppose, for example, that the health production function depends on two inputs, but the researcher can only observe one of those inputs. Only if the two inputs are neither complements nor substitutes would it be possible to estimate the marginal effect of the observed input on some transformation of the health outcome without knowledge of the level of the unobserved input or the process used to determine the choice of the second input. In general it is unlikely that the researcher would have a priori knowledge that the impacts of the unobserved inputs are separable. In this case the estimable impact of the one observed input on the health outcome would confound the ceteris paribus effect of that observed input with the effect of the unobserved input. In this paper we explore the consequences of not observing all of the relevant inputs to a health production function.

This paper uses a static model of utility maximization subject to a budget constraint in conjunction with a health production function to derive precise interpretations of estimated effects of observable inputs on health outcomes. The economic model provides considerable guidance for researchers about the types of variables that one need to include in a hybrid health production function in order to justify these interpretations. In general, these estimated effects do not correspond exactly to standard ceteris paribus marginal effects of the observed inputs in the health production function. Often the estimated effects will provide a bound on the magnitude of the true ceteris paribus effect. The economic model provides some conditions under which the estimates of the impacts of the observable inputs on health outcomes might correspond approximately to their ceteris paribus impacts. This type of analysis can be applied to any maximization problem where purchased goods might provide utility indirectly through the production of household commodities as in Lancaster (1966) or Michael and Becker (1975). We close the paper with an empirical example of how estimates of the effects of health inputs depend crucially on the inclusion or exclusion of the various forms the control variables indicated by the theoretical model.

2 Background

Early work on the estimation of production functions with missing inputs mostly focused on the case where there was a fixed unobserved input that was not varied as part of the optimization process. The motivation for these types of formulations came from an assumption that there could be unobserved, firm specific managerial factors affecting input choices and output levels (Hoch, 1955; Mundlak, 1961). In general, longitudinal data with firm or agent specific fixed effects could be used to obtain consistent estimates of the marginal impacts of the observed inputs to the production process.

Rosenzweig and Schultz (1983) took the analysis of production functions with missing inputs to a more fundamental level. In their analysis, inputs are chosen optimally as a part of a household utility maximization process, but the researcher does not observe the chosen levels for a subset of the inputs. There might also be unobserved household specific factors such as environment or genetics that correspond closely to the fixed managerial factors in the early analyses. They discuss a commonly used approach to deal with the unmeasured inputs and label this the hybrid production function. In that approach, the researcher estimates a relationship where output is a function of the observed inputs, the prices of the unobserved inputs, and the households level of exogenous income. They demonstrate that the estimated impact of an observed inputs on health outcomes in this hybrid specification do not measure the true marginal impact of the observed input holding constant the levels of the other observed inputs and the levels of the unobserved inputs. The estimated impact depends on all of the marginal impacts of the unmeasured inputs as well as the parameters of the households utility function. Unobserved inputs that are chosen as part of the households utility maximization, subject to a budget constraint, result in consequences well beyond those addressed in the early literature that only had fixed unobserved inputs affecting the choices of the variable inputs and output levels.

Todd and Wolpin (2003), in a discussion of production functions for cognitive achievement, point out that the inclusion of proxy variables like income and prices for unobserved inputs can lead to more biased measures of the impacts of the observable inputs than an empirical approach that ignores these variable that proxy for the unobserved inputs (see, also, Wolpin, 1997). They present a detailed classification of the types of approaches one might use when not all of the relevant inputs can be observed and discuss the assumptions needed with these approaches to obtain asymptotically unbiased estimates of the marginal effects of the observed inputs. They also outline several specification tests that researchers could apply to help them uncover which sets of assumptions might not be rejected by the data. A major conclusion of their study is that instrumental variables approaches will be unlikely to help resolve problems arising from omitted inputs in the production function. This happens because the omitted inputs are chosen by the families and so would typically be correlated with the observed inputs. In this situation, any instrument that has power to predict the observed input should also predict the unobserved inputs. It could not be a valid instrument. They conclude with the somewhat pessimistic advice, "It is therefore important to have data that contains a large set of inputs spanning both family and school domains."

Liu, Mroz, and Adair (2009) use a more formal derivation of Rosenzweig and Schultz's (1983) hybrid production function to explore possible biases in the estimation of marginal effects due to there being unobserved inputs. Their analysis, like the one presented in this paper, assumes that all relevant prices and incomes are observed, and they demonstrate how one can substitute conditional or rationed demand functions into the structural production function to control for the presence of the unobserved inputs. By differentiating the resulting hybrid production function with respect to the observed input, they provide an exact expression for the functional effect of observed inputs on the health production function. They also discuss the interpretation of effects when households can adjust their input decisions on a finer time scale than is reported a data set and describe the types of effects that one can estimate consistently in the presence of unmeasured inputs.

3 Preliminary Modeling Issues

A common shortcoming of the studies discussed above is their failure to provide an exact link between the theoretical model and the specification of the empirical model. In this section we fill in that gap in the literature. In the subsequent section we use the results of from this preliminary analysis to specify and interpret feasible empirical specifications of health production functions that follow from a theoretical model of a household utility optimization. Throughout the analysis in this and the subsequent section, we assume that there are only two purchased inputs used in the health production function, X and Z, and that utility depends on the amount of health produced by the household, H, and the consumption of a composite commodity C.

Let the function $H = F(X, Z)$ be the health production function. The standard demand functions for the two health inputs are given by $X = X(p_X, p_Z, p_C, I)$ and $Z = Z(p_X, p_Z, p_C, I)$ where the $p.$'s are the prices of the three purchased goods and I is exogenously determined income. Throughout this discussion we assume that one could estimate nonparametrically the two demand functions and the health production function $F(X, Z)$ if H, the two inputs X and Z, the prices of the three goods, and exogenous income I were observed by the researcher. Since the prices and income do not enter the production function directly, they are potential candidates to use as instrumental variables to control for the possible endogeneity of X and Z. The problem we want to address is what one might be able to estimate if there is only information on H, the prices, income, and the quantity of the input X. That is, the levels of input Z and consumption goods C are not observed.

A seemingly obvious approach would be to substitute the demand function for Z into the production function and then estimate this form of the hybrid production function. This demand function, by definition, will depend on the households preferences over C and H and the form of the health production function. This approach, however, will in general result in an unidentified model. This would not be an issue if one actually imposes the exact functional form of the health production function $F(X, Z)$ and has precise information about the functional forms for the demand function $Z(p_X, p_Z, p_C, I)$. To see this, substitute the demand function for the unobserved input into the production function. This yields $H = F(X, Z(p_X, p_Z, p_C, I))$. When the functional form of the demand function is unknown, this becomes $H = G(X, p_X, p_Z, p_C, I)$ where G is the hybrid production function derived using standard economic concepts. Note that the function G would likely be much less informative than the function F as all effects of X on H would need to condition on specific values of p_X, p_Z, p_C, and I, rather than on just the level of the second input Z. But without knowledge of Z or sets of strong assumptions, this might be the most one could hope to learn about the households technology for producing health.

Since the input X depends on exactly the same set of variables determining Z (i.e., each input demand is a function of p_X, p_Z, p_C, and I), there is an exact functional relationship among the five arguments in the function $G(\cdot)$. This implies that a nonparametric model for estimating the function G could admit almost any estimate of the effect of X on H through the function G by offsetting changes in the impacts of p_X, p_Z, p_C, and I on H through the function G. This is a nonparametric expression of the problem similar to perfect multicollinearity in a linear regression model. Like in the linear regression model, this identification problem can only be overcome by the imposition of some, hopefully valid, set of constraints. Economic theory, however, provides little guidance for the types of constraints one might impose in order to obtain the true impact of the input X on the health outcome.

Rosenzweig and Schultz's (1983) presentation of the hybrid production functions differs from the one presented here by its exclusion of the price of the observed input, p_X, as a determinant of the health outcome. In general this would be valid only under restrictive assumptions about the households preferences and technologies. Suppose, for example, that the households budget share of the unobserved input Z is always a constant; that is, it does not depend on p_X. This could arise from homothetic preferences for the consumption good C and heath H in the utility function and a Cobb Douglass health production function. In such situations variations in the observed input X would arise from variations in p_X, which would not be perfectly determined by variations in p_Z, p_C, and I. The Rosenzweig and Schultz formulation for the hybrid production function could more generally be derived when all households face the same price for the input X. But in this case, there would be no variation in the input X that did not arise from variations in p_Z, p_C, and I, resulting again in a non-identified specification. Without strong and mostly ad hoc assumptions, the form of the hybrid production function discussed by Rosenzweig and Schultz cannot be derived from a standard model of utility maximization.

The conditional demand function approach used by Liu et al (2009) can overcome the basic identification issue inherent in the unrestricted form of the hybrid production function G. In particular, consider the demand function for the unobserved input Z conditional on the optimally chosen level of the observed input X. Using standard rationed demand analysis, this conditional function can be written as $Z = q_c(p_C, p_Z, I^*, X)$, where $I^* = I - p_X X$ is the amount of income the household has left to allocate between consumption good C and the unobserved input Z. In general, the conditional demand for Z will depend on the amount of X chosen by the household even holding the level of I^* fixed. Substituting this constrained demand for Z into the true production function yields $H = F(X, q_c(p_C, p_Z, I^*, X))$. Without assumptions on the form of the function $q_c(\cdot)$, the estimable conditional hybrid production function becomes $H = G_C(X, p_C, p_Z, I^*)$. Provided one conditions on the remaining income I^* and the prices p_C and p_Z, as long as there is sufficient variation across households in income I and the price of the observed input, then the effect of X on H, through the function G_C and conditional on p_C, p_Z, and I^*, should be nonparametrically identified.

It is crucial that one condition on the value of I^* instead of its components in order for this particular effect of X to be identified. Liu et al's (2009) failure to do that in their empirical model likely limits one's ability to interpret their hybrid production function estimates, though many of their other estimated effects do retain a straightforward interpretation. The estimate of the partial effect of X on H obtained through the conditional hybrid production function G_C, however, does not have a simple interpretation. In the next section we derive interpretations of this type of effect using standard price theory tools.

4 Basic Model

4.1 Preferences and Technology

Assume consumers derive utility U from health H and some other consumption goods C. For simplicity, health and C are assumed to be one-dimensional. Health is produced with several inputs. We denote as X inputs which are observed and as Z the unobserved inputs. Assume preferences are given by a general utility function

$$U = U(C, H) \tag{1}$$

The household health production is given by function F with standard properties

$$H = F(X, Z), \tag{2}$$

and the household budget constraint is:

$$p_X X + p_C C + p_Z Z = I \tag{3}$$

Consider the following econometric problem. We would like to estimate the marginal product of input X on health production: $\frac{\partial F}{\partial X}$. The information available is structured in the following way. The level of X is observed; prices p_X, p_C, p_Z are observed. Income I is observed. The levels of other goods C and the health input Z are not observed. Our research goal is to understand which parameters of interest we are able to estimate and whether we can put bounds on those effects of interest.

4.2 The Equation of Interest

Consider the conditional demand function for unobserved health input Z:

$$Z = q_Z(p_C, p_Z, I - p_X X, X)$$

We assume that the data are rich enough so that we can vary X while holding the expenditure on other goods C and Z constant: $I^* = I - p_X X =$ a constant. Then, if we regress the observed health level H on the observed level of health input X (this input does not enter utility function directly) and the total expenditures on all goods other than X, I^*, the analysis of the correctly specified hybrid in the previous section implies that we we would estimate the following effect:

$$\frac{dF}{dX} = \frac{\partial F}{\partial X} + \frac{\partial F}{\partial Z}\,\frac{dZ}{dX}\bigg|_{I^*=I-p_X X=const} \tag{4}$$

The estimated effect is the sum of the effect of interest, the marginal product of input X in health production, as well as some bias related to the fact that as we change the level of input X the individual might change the level of unobserved health input Z, even when prices p_Z and p_C and expenditures on C and Z stay constant.

The question we ask is what is the direction of the bias and how large is it. Assuming that both the observed and unobserved inputs have positive marginal products, the estimated effect will be biased towards zero when the derivative of the conditional demand with respect to the observed input X is negative. To examine whether this would be the case, we need to compute how the unobservable input Z changes when we change the observed input X

holding the combined expenditure on Z and C fixed, $\frac{dZ}{dX}\Big|_{I^*=I-p_X X=const}$. In order to do that we would need to compute the derivative of the conditional demand function $Z = q_Z(p_C, p_Z, I^*, X)$ with respect to the observed input X holding I^* fixed.

4.3 Derivative of the Conditional Demand Function

Consider the following conditional maximization problem:

$$\max_{C,Z} U(C, F(X^*, Z)) \tag{5}$$

$$\text{s.t. } p_C C + p_Z Z = I^* = I - p_X X \tag{6}$$

We assume both the first order and the second order conditions for the maximization of this function hold. Compute the second order Taylor expansion for this utility function holding X and I^* constant.

$$\begin{gathered} dU = U_C dC + U_H F_Z dZ + \\ +\tfrac{1}{2}\left[U_{CC} dC^2 + 2U_{CH} F_Z dC dZ + U_{HH} F_Z^2 dZ^2 + U_H F_{ZZ} dZ^2\right] \end{gathered} \tag{7}$$

Since C and Z vary along the conditional budget constraint (**??**) the differentials dC and dZ are related by the following condition:

$$dC = -\frac{p_Z}{p_C} dZ \tag{8}$$

Zeroing out the first order term in the expansion (**??**) yields the first order condition:

$$\frac{U_H F_Z}{U_C} = \frac{p_Z}{p_C} \tag{9}$$

Second order term in the expansion above (**??**) should be non-positive. Substituting expression (**??**), which relates differentials along the budget constraint, into this term yields a second order condition:

$$\Delta \equiv U_{CC} \frac{p_Z^2}{p_C^2} - 2U_{CH} F_Z \frac{p_Z}{p_C} + U_{HH} F_Z^2 + U_H F_{ZZ} \leq 0 \tag{10}$$

Now stress this system by an infinitesimal change dX. The first order condition (**??**) can equivalently be written as:

$$p_Z U_C - p_C U_H F_Z = 0 \tag{11}$$

Totally differentiating this condition along the budget constraint (holding $I^* = I - p_X X$ constant) implies that:

$$\begin{gathered} p_Z U_{CC} dC + p_Z U_{CH} F_Z dZ - p_C U_{CH} F_Z dC - p_C U_{HH} F_Z^2 dZ - p_C U_H F_{ZZ} dZ + \\ + p_Z U_{CH} F_X dX - p_C U_{HH} F_Z F_X dX - p_C U_H F_{ZX} dX = 0 \end{gathered} \tag{12}$$

Substituting for dC from (**??**) yields:

$$\begin{gathered} \left(-U_{CC}\tfrac{p_Z^2}{p_C} + 2p_Z U_{CH} F_Z - p_C U_{HH} F_Z^2 - p_C U_H F_{ZZ}\right) dZ + \\ + \left(p_Z U_{CH} F_X - p_C U_{HH} F_Z F_X - p_C U_H F_{ZX}\right) dX = 0 \end{gathered} \tag{13}$$

Using the second order condition (**??**) in the expression above and dividing by p_C reveals that

$$\Delta dZ = dX\left(\frac{p_Z}{p_C} U_{CH} F_X - U_{HH} F_Z F_X - U_H F_{ZX}\right) \tag{14}$$

Using the first order condition (**??**) to substitute for the price ratio $\frac{p_Z}{p_C}$ yields

$$\begin{aligned} \Delta dZ &= dX\left(\tfrac{U_H F_Z}{U_C} U_{CH} F_X - U_{HH} F_Z F_X - U_H F_{ZX}\right) = \\ &= U_H dX\left(F_Z F_X\left(\tfrac{U_{CH}}{U_C} - \tfrac{U_{HH}}{U_H}\right) - F_{ZX}\right) \end{aligned} \tag{15}$$

In order for the derivative of the conditional demand to be negative $\frac{dZ}{dX}\Big|_{I - p_X X = const} \leq 0$, which from equation (**??**) would imply that the estimated effect would underestimate the true marginal effect of the observed productive health input, the following expression should be positive:

$$F_Z F_X\left(\frac{U_{CH}}{U_C} - \frac{U_{HH}}{U_H}\right) - F_{ZX} \tag{16}$$

Let's analyze this expression in more detail. First, F_X and F_Z are the marginal products of the two inputs which, for the moment, we assume are positive. F_{ZX} shows the impact of a higher level of X on the marginal product of the unobserved input Z. It is related to the substitutability between Z and X in production of health. The second term, $\frac{U_{CH}}{U_C} - \frac{U_{HH}}{U_H}$, can be given the following interpretation. Rewrite it as:

$$\frac{U_{CH}}{U_C} - \frac{U_{HH}}{U_H} = \frac{\partial}{\partial H}\left(\log \frac{U_C}{U_H}\right) \tag{17}$$

This expression tells us how marginal rate of substitution between health and consumption changes if we fix C and move across indifference curves changing H. If both C and H are normal goods then this term will be positive, or at least non-negative. Normality of just one of them might be sufficient for this term to be positive. This establishes the following result:

Theorem 1. *Suppose health H and other goods C are normal goods and the degree of complementarity in health production between observed input X and unobserved input Z is sufficiently small (the cross derivative F_{ZX} is small if positive or negative: i.e. the increase in one of the inputs lowers the marginal effect of the other). Then the regression of observed health H on observed health input X holding prices p_C, p_Z and total expenditure on C and Z ($I^* = I - p_X X$) constant, would underestimate (or perhaps even cause to become negative) the true value of the marginal product of X in health production.*

This theorem provides key information for interpreting the estimated "effect" of the observed input X on health H using a hybrid production function, as expressed in equation (**??**).

4.4 An Unobserved Input with a Negative Impact

Our discussion so far implicitly assumed that Z has a positive impact on health: $\frac{\partial F}{\partial Z} \geq 0$. In empirical applications it is also important to consider inputs which decrease health: smoking, alcohol etc. It is easy to verify though that the main result (Theorem **??**) still applies in this case. Indeed the bias in estimated coefficient on X in the equation (**??**) is given by:

$$Bias = \frac{\partial F}{\partial Z} \frac{dZ}{dX}\bigg|_{I-p_X X=const} \tag{18}$$

Using equation (**??**), this bias will be negative provided the following condition holds:

$$F_Z^2 F_X \frac{\partial}{\partial H}\left(\log \frac{U_C}{U_H}\right) - F_Z F_{ZX} \geq 0 \tag{19}$$

In this case we would need F_{ZX} to be positive, or small in absolute value if negative, for Theorem **??** to hold[1]. Positive F_{ZX} means that higher values of X increase F_Z. Since $F_Z < 0$ this means that higher values of X decrease the damage from missing input Z.

One example of this relationship which we pursue in the empirical part of the paper is dental cleaning and smoking. If dental cleaning reduces the damage from smoking then omission of smoking from the regression (while properly controlling for expenditures I^*) would result in a negative bias for the estimated marginal effect of dental cleaning.

[1]In the case when $F_Z < 0$ it is not clear why consumer would buy any quantity of this input. In order to have $Z > 0$ as a result of consumer maximization Z should have some direct impact on utility. This is what we analyze in the next section.

4.5 Dual Impacts of the Unobserved Input

Consider the case when the unobserved input Z has an impact on utility through two channels. As before it affects individual health outcome. Now we also allow Z to influence the utility function directly. For example, Z might be exercise or smoking. Both of these affects health, but people could also receive direct utility from their consumptions.

In this case utility function will be:

$$U = U(C, Z, H) \tag{20}$$

with the same budget constraint as before. Taking a second order Taylor expansion reveals

$$\begin{gathered} dU = U_C dC + (U_Z + U_H F_Z) dZ + \\ \frac{1}{2}\left[U_{CC} dC^2 + 2(U_{CZ} + U_{CH} F_Z) dC dZ + \right. \\ \left. +(U_{ZZ} + 2U_{ZH} F_Z + U_{HH} F_Z^2 + U_H F_{ZZ}) dZ^2\right] \end{gathered} \tag{21}$$

The differentials dC and dZ are again taken along the conditional budget constraint and hence are related as:

$$dC = -\frac{p_Z}{p_C} dZ \tag{22}$$

Substituting relation (**??**) into the expansion (**??**) above yields the first order condition

$$p_Z U_C - p_C (U_H F_Z + U_Z) = 0 \tag{23}$$

with the second order condition being

$$\Delta_2 \equiv U_{CC} \frac{p_Z^2}{p_C^2} - 2(U_{CZ} + U_{CH} F_Z) \frac{p_Z}{p_C} + (U_{ZZ} + 2U_{ZH} F_Z + U_{HH} F_Z^2 + U_H F_{ZZ}) \leq 0 \tag{24}$$

As before consider stressing the system with some $dX > 0$, holding total expenditure on C and Z constant. Totally differentiating the first order condition (**??**) yields:

$$\begin{aligned} &p_Z U_{CC} dC + p_Z(U_{CZ} + U_{CH}F_Z)dZ - p_C(U_{CZ} + U_{HC}F_Z)dC- \\ &-p_C(U_{ZZ} + 2U_{ZH}F_Z + U_{HH}F_Z^2 + U_H F_{ZZ})dZ+ \\ &+\left(p_Z U_{CH}F_X - p_C(U_{HH}F_Z F_X + U_H F_{ZX} + U_{ZH}F_X)\right)dX = 0 \end{aligned} \tag{25}$$

Substituting the conditional budget constraint differential (**??**) in the equation above yields

$$\begin{aligned} &\left(\tfrac{p_Z^2}{p_C^2}U_{CC} - 2\tfrac{p_Z}{p_C}(U_{CZ} + U_{CH}F_Z) + U_{ZZ}+\right. \\ &\left.+2U_{ZH}F_Z + U_{HH}F_Z^2 + U_H F_{ZZ}\right)dZ = \\ &= \left(\tfrac{p_Z}{p_C}U_{CH}F_X - (U_{HH}F_Z F_X + U_H F_{ZX} + U_{ZH}F_X)\right)dX \end{aligned} \tag{26}$$

or, equivalently,

$$\Delta_2 dZ = \left(\tfrac{p_Z}{p_C}U_{CH}F_X - (U_{HH}F_Z F_X + U_H F_{ZX} + U_{ZH}F_X)\right)dX \tag{27}$$

From first order condition (**??**) the price ratio $\frac{p_Z}{p_C}$ can be expressed as

$$\frac{p_Z}{p_C} = \frac{U_H F_Z + U_Z}{U_C} \tag{28}$$

Substituting (**??**) into expression (**??**) above results in:

$$\tfrac{\Delta_2 dZ}{dX} = U_H\left(F_X F_Z\left(\tfrac{U_{CH}}{U_C} - \tfrac{U_{HH}}{U_H}\right) - F_{ZX}\right) + U_Z F_X\left(\tfrac{U_{CH}}{U_C} - \tfrac{U_{ZH}}{U_Z}\right) \tag{29}$$

Total bias $\frac{\partial F}{\partial Z}\frac{dZ}{dX}\Big|_{I^*=I-p_X X=const}$ can be written then as:

$$Bias = F_Z\frac{U_H\left(F_X F_Z\left(\tfrac{U_{CH}}{U_C} - \tfrac{U_{HH}}{U_H}\right) - F_{ZX}\right) + U_Z F_X\left(\tfrac{U_{CH}}{U_C} - \tfrac{U_{ZH}}{U_Z}\right)}{\Delta_2} \tag{30}$$

This bias consist of two terms the first one is similar to what we had in the case when omitted health input was not have any effect on utility (see e.g. (**??**)):

$$Bias_1 = \frac{U_H \left(F_X F_Z^2 \left(\frac{U_{CH}}{U_C} - \frac{U_{HH}}{U_H} \right) - F_Z F_{ZX} \right)}{\Delta_2} \tag{31}$$

As before $\frac{U_{CH}}{U_C} - \frac{U_{HH}}{U_H} = \frac{\partial}{\partial H} \left(\log \frac{U_C}{U_H} \right) \geq 0$ from normality of C and H and the analysis goes in the same way as in previous section to establish the conditions under which $Bias_1$ will be negative.

The second term in the bias is:

$$Bias_2 = \frac{U_Z F_Z F_X \left(\frac{U_{CH}}{U_C} - \frac{U_{ZH}}{U_Z} \right)}{\Delta_2} = \frac{U_Z F_Z F_X}{\Delta_2} \frac{\partial}{\partial H} \left(\log \frac{U_C}{U_Z} \right) \tag{32}$$

The direction of the bias depends on whether this term is negative or positive. The sign of the derivative $\frac{\partial}{\partial H} \left(\log \frac{U_C}{U_Z} \right)$ is related to the degree of substitutability/complementarity of C and Z as consumption goods as the level of health increases. We have not yet obtained a simple interpretation for this term in the general case. However, we are able to sign it in the following special case.

Assume that utility from consuming goods, which are not health inputs, C is independent of the level of health H: $U_{CH} = 0$, i.e. marginal utility of reading a book is not affected by how healthy/sick a person is. In this case in order to get negative bias result we should have $F_Z \left(\frac{U_{CH}}{U_C} - \frac{U_{ZH}}{U_Z} \right) \geq 0$[2]. Then the direction of the bias will be determined by $F_Z U_{ZH}$. If $F_Z < 0$, i.e. Z is some health input which people enjoy consuming but at the same time it

[2] $\Delta_2 \leq 0$ from second order conditions and we assume that $U_Z > 0$ and $F_X > 0$, i.e. the observed input is beneficial for the health and utility we receive from unobserved health input is positive, i.e. Z can be interpreted as smoking.

damages their health, then we would need that marginal utility of consuming this input U_Z to increase in the level of health H: $U_{ZH} \geq 0$. Essentially this amounts to assuming that people do not enjoy smoking as much when they get sick, i.e. they do not have enough health to tolerate/afford the higher level of smoking.

In general case when $U_{CH} \neq 0$ for the $Bias_2$ to be negative we would either need that either $\frac{\partial}{\partial H}\left(\log \frac{U_C}{U_Z}\right) F_Z \geq 0$ or that the other term $Bias_1$ which is likely to be negative to dominate in the sum.

To conclude the omission of an essential health input is likely to result in downward bias in the estimated marginal product of health inputs when one uses a correctly specified hybrid production function. That function must include as explanatory variables the prices of the omitted inputs and consumption goods as well as the total expenditure on consumption goods and omitted inputs.

5 Empirical Evidence

5.1 Data Description

We use data from Behavioral Risk Factor Surveillance System (BRFSS) survey conducted by Center of Disease Control (CDC) for the year of 2008. This is a comprehensive dataset on health outcomes (physical health, mental well being, bmi, disability, and the incidence of several diseases) as well as possible health inputs such as visits to physicians, dentists, eye exams, smoking, and alcohol consumption. We focus on prime aged married white males who do not appear to suffer from debilitating illnesses that might affect their ability to work. Our primary outcome variable is a self-reported health measure.

This dataset codes health status on a 1-5 scale with one being excellent and 5 being poor. To define our health outcome variable, we invert this scale so that higher values correspond to better health.

We consider three health inputs: dental services, alcohol consumption and tobacco smoking. The dental services variable measures how recently a person had his last dental cleaning (in years), higher values correspond to more recent dental cleaning. We use the number of times a person binge drinks per month (defined as the consumption of more than 5 drinks on one occasion) as the measure of alcohol consumption. Tobacco smoking is categorical variable that shows whether a person smokes often, occasionally or not at all. We assume that person who smokes occasionally consumes 7.5 packs per month, whereas person who smokes often consumes 30 packs per month. The survey records each respondent's income category (below \$10,000; \$10,000 to \$15,000; \$15,000 to \$20,000; \$20,000 to \$25,000; \$25,000 to \$35,000; \$35,000 to \$50,000; \$50,000 to \$75,000; and \$75,000 or more). We use this categorical variable to construct our income measure by imputing for each individual the midpoint of his income category. The lack of a more detailed income measure is the primary shortcoming of this dataset. We merge the BRFSS dataset with the region level data on prices of dentists, beer, wine, and cigarette taxes. We also include other prices such as apartment rent and total energy costs. Table 1 contains summary statistics for all variables used in our analysis.

5.2 Estimation Results

Following the theoretical results of the previous section we consider a regression model with all three inputs in health production present as well as the total expenditure available for all other health inputs and goods.

$$H_i = \alpha + \beta_1 Dental_i + \beta_2 Drinking_i + \beta_3 Smoking_i + \gamma Income_i^* \qquad (33)$$

Here $Income_i^* = Income_i - Drinking_i P_{Drink} - Dental_i P_{Dent} - Smoking_i P_{Cigs}$ measures the income spent on all other goods except three health goods under consideration. We also include age, the square of age and the individual's education level as control variables. Our goal in this empirical analysis is to take this specification as a baseline and explore how the estimated impacts of the health inputs change when we omit one of the inputs to simulate an unobserved health determinant. Recall that the theoretical model implies that the estimated impact of the observed productive input should provide a lower bound on the true marginal product (or exhibit a negative bias from the true marginal product) when the empirical model excludes an input but correctly includes its price and the household's total income spent on all goods other than the included health inputs.

Since health inputs as well as income might be endogenous, in this preliminary analysis we estimate this regression by 2SLS with Dentist Services, Alcohol, Smoking and $Income^*$ being instrumented by beer, wine, dentist prices, cigarette taxes, apartment rent, total energy costs, and cost of haircut. In future work we will explore treating total income as exogenous and other possible ways to instrument for income. Note that the economic model implies that the prices of the three included health inputs should be valid instruments for these variables.

We start our analysis with the sample of men aged 25 to 35. The estimated coefficients for this sample are in Table 2. The estimates in the first column of Table 2 do not include the income measure while those in the second column do include the income measure as suggested by the economic model. Surprisingly, the estimated impacts of the three health variables barely change when we include income for other expenditures as a regressor. This could possibly be interpreted as specification test for a completely specified health production function. A more comprehensive test, however, would include the prices of all other goods as regressors. We see that dental cleaning indeed improves the reported health outcomes, whereas both drinking and smoking seem to have negative (though not significant) impacts on it.

To simulate the effect of having an unobserved health input, we drop the smoking variable from the production function. We then examine how the estimated effects of the other two health inputs, dentists visits and binge drinking, change as a consequence of not observing a key health input. Following the methodology outlined above we also include price of cigarettes P_{cigs} and adjusted income available to spend on all other goods except the two included health inputs, $Income^{**} = Income_i - Drinking_i P_{Drink} - Dental_i P_{Dent}$, in the regression.

$$H_i = \alpha + \beta_1 Dental_i + \beta_2 Drinking_i + \gamma Income_i^{**} + \delta P_{cigs} \qquad (34)$$

We estimate this regression again by 2SLS with the same set of instrumental variables as before. Estimation results are presented in the third column of Table 2. We see that omission of smoking reduces coefficient on dental cleaning. This is precisely the type of effect that was suggested by the theoretical model. The positive effect of the cigarette tax variable on health in column 3 suggests that smoking is a bad input, provided that cigarette consumption is a normal good in its conditional demand. The coefficient on alcohol consumption (binge drinking) also becomes more negative. If it were known that increased alcohol consumption truly contributes to bad self-reported health, then this change would be at odds with the results of the theoretical model. One could, however, use the theoretical analysis to interpret this type of change to indicate that alcohol consumption is a productive health input. The magnitude of this change, however, is substantively small and the estimated effect is not statistically different from zero.

The remaining columns in Table 2 represent robustness checks. In the column 4 we alter the theoretically correct specification (in column 3) by excluding the price of the now "unobserved" input (smoking), the cigarette tax. The estimate of the effect of dental visits becomes larger than that in the baseline specification (column 2). If one were to interpret this dental visit effect in the context of the theoretical model, one would incorrectly conclude that the lower bound for the effect of dental visits is almost fifty percent higher than what is suggested by the correctly specified regression in column 3. However it is difficult to interpret this as it would not make sense from an economic modeling perspective unless all individuals face the same taxes on cigarettes, which we know is false. The coefficient for the effect of alcohol consumption also becomes less negative compared to its effect in the correctly specified specification 3. In specification 5 we include the price of cigarettes but exclude the adjusted income measure. The estimated "lower bound" for dental visits also rises in this misspecified model compared to the correctly specified regression results reported in column 3. The regression model whose results are in column 6 correspond to what one would estimate if she believed she had included all of the relevant inputs to the health production function but in fact was missing an input. The effects of dental visit and drinking estimated from this specification are the most positive of any reported in Table 2. While the correctly specified lower bounds found in column (3) do not necessarily rule out effects of these magnitudes, these estimated effects cannot be interpreted as marginal products in the health production function. In addition, the effects for specification (6) are more positive than those in the baseline specification.

For further robustness checks, we expand the age range of the sample from 36 to 44 to ages 26 through 54. In Table 3 we report estimates for specifications identical to those in Table 2 but for the larger age range pr respondents. In general, the results for this expanded age range mimic those found in Table 2. One key exception is the statistical significance of the income effect in specification 2. This suggests that the health production function depends on more than just the three inputs we consider here. When we exclude smoking as a health determinant but include its price (the cigarette tax) and the adjusted income, the effect of dental visits becomes more negative as suggested by the economic model. The coefficient on alcohol consumption also becomes more negative. Failing to include either the price variable (specification 4) or the adjusted income variable (specification 5) results in both the effects of dental visits and drinking to become more positive. Assuming that dental visits and drinking are the only two health inputs (specification 6) again provides the most positive estimates for these effects for any of the specifications we considered. Since the estimates in column 3 provide the ”full information” bounds on the effects of these two inputs when a key health input is unmeasured, it is not clear how one should interpret the effects in any of the specifications 4, 5, or 6.

6 Summary

This paper demonstrates the power of simple economic theory to help a researcher specify empirical models of health production functions and interpret effects estimated using these correctly specified hybrid production functions. Provided observed and unobserved health inputs are not strongly complementary, the theoretical analysis reveals that the estimated effect of an observed productive input would actually be an estimate of a lower bound on the marginal product of the observed health input. For "bad" observed inputs (e.g., smoking), one would estimate an upper bound on its true effect, which corresponds to a lower bound on the magnitude of it deleterious effect. Our empirical analysis using a self-reported health measure and information on smoking, dental visits and binge drinking using BRFSS data generally support the implications of the theoretical analysis. When we exclude smoking from the health production function, the estimated impact of the benefits from having one's teeth cleaned falls as implied by the empirical model. The corresponding effect on the estimate of the impact of binge drinking,however, would only be in accord with the theoretical model if a higher level of binge drinking would lead to a higher level of self-reported health status. The magnitudes of these alcohol consumption effects, however, are quite small.

Both the theoretical analysis and the empirical investigation need to be enhanced substantially. For our "simulation" of the effect of excluding a health input, we would need to derive the theoretical implications of having one fewer observed inputs rather than just one unobserved health input. In addition, many relevant health inputs influence utility directly as well as indirectly through the health production function. We need to extend the theoretical analysis to find interpretations of the empirical estimates of these types of health inputs. The BRFSS data set we use in this preliminary analysis does provide fairly large sample sizes, but the income measure in this data set is fairly crude. The measures for the health inputs we examine are also fairly imprecise, but it is unlikely that other existing data sources would provide significantly better measurements. We need to explore whether alternative data sources would also provide empirical support for the type of guidance provided by our theoretical analysis.

The sensitivity of the point estimates we obtain to the inclusion or the exclusion of the prices of the "unmeasured health inputs" and to the inclusion or exclusion of the expenditure allocated to the excluded inputs suggests that the types of theoretical issues we raise could have important substantive implications for health policy research. Recognizing that we almost never observe all of the relevant inputs to a health production function has key implications about the types of variables one needs to incorporate in empirical analyses. It also requires researchers to interpret estimated effects not as actual marginal effects but as bounds on the marginal effects. In addition, the theoretical framework suggests that just collecting information on health inputs will usually not be sufficient for researchers to obtain interpretable effects of health inputs on health outcomes. Whenever households choose the levels of some health inputs that are not measured in a data set, it is crucial that one control for the total expenditures on all of the observable health inputs.

7 References

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Table 1: Summary Statistics.

Variable	N-obs	Mean	st. dev.	Min	Max
Subsample of males ages 35-45					
Dentist Visits	2450	2.596	0.858	0.000	3.000
Alcohol	2450	0.829	2.647	0.000	30.000
Smoking	2450	0.204	0.581	0.000	2.000
Income* (in $10,000)	2450	8.340	2.501	0.470	10.000
Income** (in $10,000)	2450	8.341	2.500	0.480	10.000
Age	2450	40.213	2.559	36.000	44.000
Age-Squared/100	2450	16.236	2.053	12.960	19.360
Education	2450	3.301	0.890	1.000	4.000
Cigarette Tax	2450	1.169	0.660	0.070	2.575
Apartment Rent	2450	976.774	422.210	462.000	3475.000
Total Energy Costs	2450	191.514	45.846	113.530	328.810
Price of Haircut	2450	13.923	2.535	8.210	25.080
Subsample of males ages 25-55					
Dental Cleaning	7151	2.603	0.846	0.000	3.000
Alcohol	7151	0.834	2.818	0.000	30.000
Smoking	7151	0.220	0.597	0.000	2.000
Income* (in $10,000)	7151	8.243	2.535	0.470	10.000
Income** (in $10,000)	7151	8.244	2.534	0.480	10.000
Age	7151	42.477	7.516	26.000	54.000
Age-Squared/100	7151	18.607	6.253	6.760	29.160
Education	7151	3.280	0.884	1.000	4.000
Cigaret Tax	7151	1.152	0.649	0.070	2.575
Apartment Rent	7151	954.777	398.142	462.000	3475.000
Total Energy Costs	7151	189.474	44.544	113.530	328.810
Price of Haircut	7151	13.844	2.468	8.210	25.080

Table 2: Sample of white married males age 25-55.

	(1)	(2)	(3)	(4)	(5)	(6)
Dental Cleaning	0.463***	0.192*	-0.040	0.484***	0.173	-0.052
	(0.094)	(0.112)	(0.128)	(0.084)	(0.107)	(0.125)
Alcohol	-0.006	-0.086**	-0.068*	-0.008	-0.067*	-0.052
	(0.035)	(0.039)	(0.040)	(0.035)	(0.035)	(0.037)
Smoking	-0.073	0.246	0.156			
	(0.157)	(0.171)	(0.179)			
$Income^*$		0.035***	0.040***		0.030***	0.037***
(in $ 10,000)		(0.008)	(0.008)		(0.007)	(0.007)
Age	-0.030*	-0.038**	-0.044***	-0.029*	-0.039**	-0.045***
	(0.016)	(0.016)	(0.016)	(0.016)	(0.016)	(0.015)
Age-squared/100	0.028	0.036*	0.046**	0.027	0.038**	0.047**
	(0.020)	(0.019)	(0.019)	(0.020)	(0.019)	(0.019)
Education	0.111***	0.128***	0.138***	0.119***	0.103***	0.124***
	(0.027)	(0.027)	(0.027)	(0.021)	(0.020)	(0.021)
Cigarette Tax						-0.009
						(0.021)
Constant	-2.672***	-2.134***	-2.132***	-2.776***	-1.922***	-2.048***
	(0.436)	(0.448)	(0.472)	(0.378)	(0.407)	(0.453)
Observations	5,534	5,534	5,534	5,534	5,534	5,534

Dependent variable in all regressions is health status. Sample includes all white "healthy" married males aged between 25 and 55 at the moment of the survey. $Income^*$ in specifications (2) and (3) is household income net of spending on three health inputs included in the regression. In specifications (5) and (6) $Income^*$ is household income (in $10,000) net of spending on two health inputs included in the regression. Specifications (3) and (6) include price indices for groceries, housing, utilities. Standard errors are in parentheses. ***, **, * indicate significance at 1%, 5%, and 10% respectively.

Table 3: Sample of white married males age 25-55.

	(1)	(2)	(3)	(4)	(5)	(6)
Dental Cleaning				0.461***	0.229**	-0.034
				(0.093)	(0.106)	(0.124)
Alcohol	0.020	-0.096**	-0.066			
	(0.033)	(0.039)	(0.040)			
Smoking	-0.434***	0.211	0.159	-0.077	0.117	0.028
	(0.132)	(0.170)	(0.175)	(0.154)	(0.154)	(0.161)
*Income**		0.043***	0.039***		0.027***	0.034***
(in $10,000)		(0.007)	(0.007)		(0.007)	(0.007)
Age	-0.032**	-0.040**	-0.043***	-0.030*	-0.036**	-0.043***
	(0.016)	(0.016)	(0.016)	(0.016)	(0.015)	(0.015)
Age-squared/100	0.035*	0.040**	0.045**	0.028	0.036*	0.047**
	(0.019)	(0.019)	(0.019)	(0.020)	(0.019)	(0.019)
Education	0.117***	0.133***	0.136***	0.113***	0.142***	0.148***
	(0.026)	(0.027)	(0.026)	(0.025)	(0.025)	(0.025)
Price of Dentist			-0.001			
			(0.001)			
Price of Beer						-0.006
						(0.025)
Constant	-1.478***	-1.675***	-2.225***	-2.677***	-2.322***	-2.216***
	(0.347)	(0.359)	(0.414)	(0.435)	(0.422)	(0.506)
Observations	5,534	5,534	5,534	5,534	5,534	5,534

Dependent variable in all regressions is health status. Sample includes all white "healthy" married males aged between 25 and 55 at the moment of the survey. *Income** in specifications (2), (3), (5) and (6) is household income net of spending on three health inputs included in the regression. Specifications (3) and (6) include price indices for groceries, housing, utilities. Standard errors are in parentheses. ***, **, * indicate significance at 1%, 5%, and 10% respectively.

Table 4: Sample of white married males age 35-45.

	(1)	(2)	(3)	(4)	(5)	(6)
Dental Cleaning	0.315***	0.214**	0.062	0.333***	0.212**	0.073
	(0.101)	(0.101)	(0.116)	(0.100)	(0.101)	(0.115)
Alcohol	0.018	-0.013	-0.021	0.009	-0.011	-0.020
	(0.040)	(0.039)	(0.039)	(0.039)	(0.038)	(0.038)
Smoking	-0.164	0.023	-0.013			
	(0.164)	(0.166)	(0.166)			
Income*		0.038***	0.040***		0.038***	0.039***
(in $10,000)		(0.009)	(0.009)		(0.009)	(0.009)
Age	-0.055	-0.056	-0.067	-0.035	-0.058	-0.064
	(0.248)	(0.241)	(0.238)	(0.248)	(0.240)	(0.237)
Age-squared/100	0.050	0.051	0.068	0.025	0.054	0.065
	(0.309)	(0.301)	(0.296)	(0.309)	(0.299)	(0.295)
Education	0.114***	0.101***	0.105***	0.135***	0.099***	0.106***
	(0.034)	(0.034)	(0.033)	(0.027)	(0.028)	(0.027)
Cigarette Tax						-0.030
						(0.035)
Constant	-1.639	-1.657	-2.048	-2.170	-1.593	-2.210
	(4.968)	(4.834)	(4.768)	(4.956)	(4.802)	(4.742)
Observations	1,938	1,938	1,938	1,938	1,938	1,938

Dependent variable in all regressions is health status. Sample includes all white "healthy" married males aged between 35 and 45 at the moment of the survey. *Income** in specifications (2) and (3) is household income net of spending on three health inputs included in the regression. In specifications (5) and (6) *Income** is household income (in $10,000) net of spending on two health inputs included in the regression. Specifications (3) and (6) include price indices for groceries, housing, utilities. Standard errors are in parentheses. ***, **, * indicate significance at 1%, 5%, and 10% respectively.

Table 5: Sample of white married males age 35-45.

	(1)	(2)	(3)	(4)	(5)	(6)
Dental Cleaning				0.319***	0.213**	0.062
				(0.101)	(0.101)	(0.116)
Alcohol	0.027	-0.011	-0.018			
	(0.039)	(0.039)	(0.039)			
Smoking	-0.252	-0.010	-0.010	-0.149	0.009	-0.034
	(0.157)	(0.163)	(0.166)	(0.159)	(0.160)	(0.163)
*Income**		0.043***	0.042***		0.037***	0.039***
(in $10,000)		(0.009)	(0.009)		(0.009)	(0.009)
Age	-0.059	-0.059	-0.068	-0.048	-0.061	-0.075
	(0.242)	(0.238)	(0.238)	(0.247)	(0.241)	(0.237)
Age-squared/100	0.061	0.059	0.071	0.041	0.057	0.078
	(0.302)	(0.297)	(0.297)	(0.308)	(0.300)	(0.296)
Education	0.133***	0.112***	0.111***	0.110***	0.104***	0.111***
	(0.033)	(0.033)	(0.033)	(0.033)	(0.032)	(0.032)
Price of Dentist			-0.002			
			(0.002)			
Price of Beer						0.002
						(0.043)
Constant	-0.887	-1.181	-1.997	-1.774	-1.560	-1.912
	(4.844)	(4.774)	(4.769)	(4.947)	(4.823)	(4.767)
Observations	1,938	1,938	1,938	1,938	1,938	1,938

Dependent variable in all regressions is health status. Sample includes all white "healthy" married males aged between 25 and 55 at the moment of the survey. *Income** in specifications (2), (3), (5) and (6) is household income net of spending on health inputs included in the corresponding regression. Specifications (3) and (6) include price indices for groceries, housing, utilities as controls. Standard errors are in parentheses. ***, **, * indicate significance at 1%, 5%, and 10% respectively.