A Simple Test for Spurious Regression

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Spurious Regression since the roaring twenties

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- Granger and Newbold(1974) reappraised this phenomenon in LS. See examples of Plosser and Schwert (1978) Hendry (1980).
- Phillips (1986) introduced the asymptotics of non-stationary processes and explained previous results using driftless $I(1)$ processes.
- Park and Phillips (1989) proved Spurious Regression (SR) with $I(2)$ processes; Marmol (1996) did it with variables integrated of different orders; Entorf (1997) used $I(1)$ plus drift processes.
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Spurious Regression pervades everything...

- Stationary series: Granger et al (2001), Mikosch and Starica (2004), Mikosch and de Vries (2006).
- Trend Stationarity: Hasseler (1996,2000, 2003), Kim et al (2004), NVS (2006,2007).
- Statistical tests: Jarque-Bera [Breusch-Pagan-Godfrey Giles, 2007], Granger-Causality [VSVV, 2008], Ramsey-RESET, MCLeod and Li, Keenan [LKN, 2005].
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In other words:

When you estimate by LS...

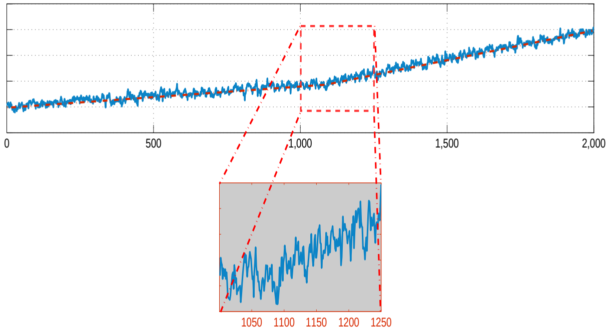
$$y_t = \alpha + \beta x_t + u_t$$

The regression may be spurious when the variables are...

In other words:

Broken trend stationary

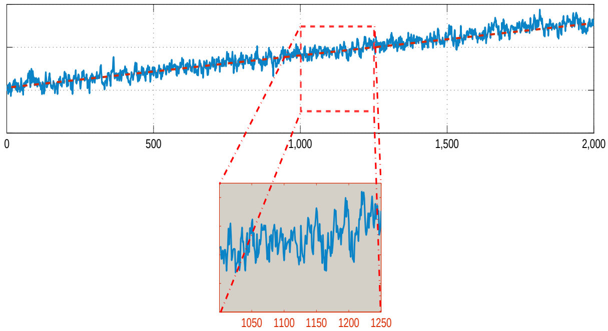
Figure: BTS



In other words:

Trend Stationary

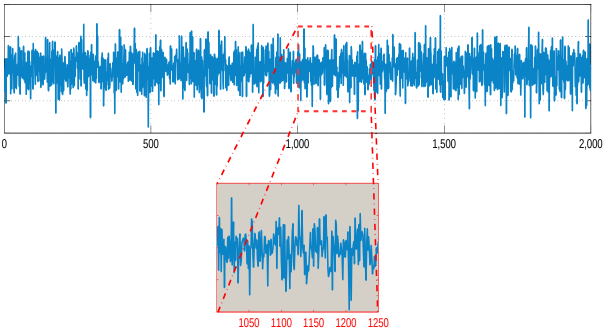
Figure: TS



In other words:

Stationary

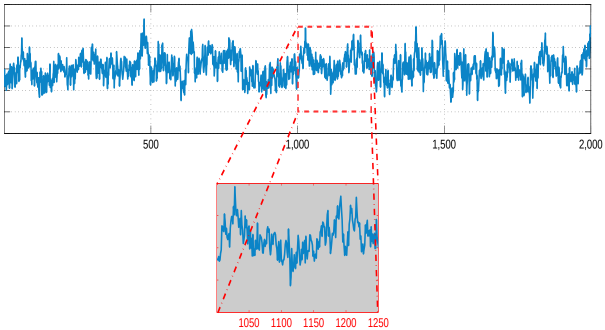
Figure: MA(q)



In other words:

Stationary fractionally integrated

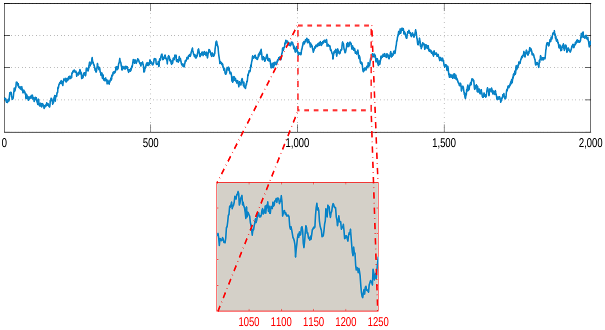
Figure: stationary ARFIMA



In other words:

Driftless Unit Root

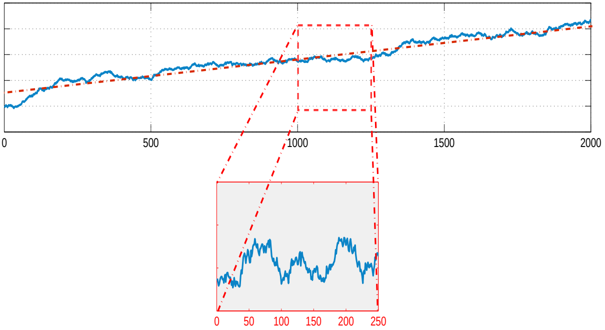
Figure: I(1)



In other words:

Unit Root

Figure: I(1)+drift



Why are these DGPs interesting (examples)?

Income Convergence: Bernard and Durlauf(1996)

If the–log–per capita income gap between two economies has...

- ...a Unit Root: Divergence
- ...a negative (positive) Deterministic Trend: Catching-up (Lagging-behind)
- ... none of them: Convergence
- ...both kinds of Trends: Loose Catching-up or Loose Lagging-behind

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1. Inflation Targeting (Gregoriou and Kontonikas, 2006): If the targeting monetary policy is working, the inflation series should follow a Broken–level mean stationary process.

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4. Purchasing Power Parity (Wallace and Shelley, 2005): If the PPP hypothesis is true, RER should be stationary or trend-stationary:

What to do if one fears spurious regression?

Pre-testing...

ANY TIME-SERIES ECONOMETRICS ANALYSIS SHOULD BEGIN WITH A VAST PLETHORA OF STATISTICAL TESTS IN ORDER TO OBTAIN AS MUCH AS POSSIBLE INFORMATION ON THE TRENDING MECHANISM OF THE VARIABLES. BASED ON THIS EVIDENCE, MANY PROCEDURES ARE THEN AVAILABLE...

What to do if one fears spurious regression?

Unit Root test

DF [Dickey-Fuller (1979,1981)]; KPSS [Kwiatkowski et al (1992)]; GLS-detrended [Elliott et al (1996)]; PP [Phillips-Perron (1988); Ng and Perron (1996)].

What to do if one fears spurious regression?

Unit roots and Breaks

PERRON (1989); ZIVOT AND ANDREWS (1992); LUMDAINE AND PAPELL (1997); PERRON (1997); CARRION-I-SILVESTRE AND SANSÒ (2006). SEE ALSO BAI AND PERRON (1998) FOR DETERMINISTIC BREAKS EXCLUSIVELY.

What to do if one fears spurious regression?

Long Memory

Lo (1991); Liu et al (1993); Robinson (1994); Lobato and Robinson (1996); Aue et al (2008).

Cures for spurious regression:

Cures for spurious regression:

(i) (ii) (iii) Stochastic trending mechanisms:

1. Engle-Granger Cointegration test.
2. Error Correction Models.
3. Johansen Cointegration Test.

Cures for spurious regression:

(i) (ii) (iii) Deterministic trending mechanisms:

1. Co-trending
2. Co-breaking

Cures for spurious regression:

(i) (ii) (iii) Use the consistent t-ratio suggested by Sun (2004):

1. Inspired in the HAC estimate of the variance-covariance matrix,
2. The truncation parameter equals the sample size.

Scope of this work

- To introduce a test that distinguishes spurious regressions from cointegrated/co-trended ones.
  1. We present the procedure of the test.
  2. We provide the asymptotic arguments that underlie the test.
  3. We provide critical values.
  4. We finally present finite-sample evidence.

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Considered DGPs:

We circumscribe to the following DGPs:

Case	Name	Model
1	$I(1)$	$z_t = z_{t-1} + u_{zt}$
2	$I(1) + dr$	$z_t = \mu_z + z_{t-1} + u_{zt}$
3	TS	$z_t = \mu_z + \beta_z t + u_{zt}$
4	CS^*	$y_t = \mu_y + \beta_y x_t + u_{yt}$

Table: $z_t = y_t, x_t$. *: CS accounts for "Correct Specification".

Procedure: the new test is builded as follows:

Regress each variable, x_t and y_t, on a constant and a deterministic trend:

$$z_t = \delta_z + \gamma_z t + u_{zt} \quad (1)$$

where $z = x, y$.

Build the residuals $\hat{u}_{xt}$ and $\hat{u}_{yt}$ and use them to estimate:

$$\hat{u}_{yt} = \alpha_f + \beta_f \hat{u}_{xt} + \epsilon_t \quad (2)$$

Procedure: the new test is builded as follows:

The pre-testing evidence must be considered

Calculate the standard t-ratio associated to $\hat{\beta}_f$ and, if there is prior evidence of unit root in both variables, divide the t-ratio by $\sqrt{T}$; otherwise do not normalize it; we will refer to the resulting pseudo t-statistic as $\tau^{w}_{\hat{\beta}_f}$, where $w = r$ means rescaled and $w = n$ means not re-scaled.

Procedure: the new test is builded as follows:

Perform the test:

$\mathcal{H}_0 : \ \beta = 0$ against the alternative hypothesis $\mathcal{H}_a : \ \beta \neq 0$. The asymptotic distribution of $\tau^{W}_{\hat{\beta}_f}$ under the null hypothesis is non-standard, but does not include any perturbation parameter and can be therefore tabulated. Under the alternative hypothesis, $\tau^{W}_{\hat{\beta}_f} = O_p(T^{\nu})$, where $\nu \geq \frac{1}{2}$.

First step (Deterministic Regression) $z_t = \delta_z + \gamma_z t + u_{zt}$

demo

Proposition

Let x_t and y_t be generated by DGPs i and j for $i, j = 1, 2, 3$, and 4 in table 1 and use them to estimate specification (1). The order in probability of the t-ratio, $t_{\hat{\gamma}_z}$, for $z = y, x$, are:

- $t_{\hat{\gamma}_z} = O_p\left(\sqrt{T}\right)$ *for z generated as in DGP (1) or y generated as in DGP (4) and x as in DGP (1),*
- $t_{\hat{\gamma}_z} = O_p\left(T\right)$ *for z generated as in DGP (2) or y generated as in DGP (4) and x as in DGP (2),*
- $t_{\hat{\gamma}_z} = O_p\left(T^{\frac{3}{2}}\right)$ *for z generated as in (3) or y generated as in DGP (4) and x as in DGP (3),*

FIRST STEP (DETERMINISTIC REGRESSION) $z_t = \delta_z + \gamma_z t + u_{zt}$

demo

Evidence of a deterministic trend

Note that the previous result allows one to infer about the presence of a deterministic trend! This idea has already been explored by GVS(2008).

First step (Deterministic Regression) $z_t = \delta_z + \gamma_z t + u_{zt}$

demo

Corollary

Let: [1] y_t be generated by DGP (1); [2] y_t and x_t be generated by DGPs (4) and (1), respectively. In both cases use y_t to estimate equation (1). Then the re-scaled t-ratio associated to $\hat{\gamma}_z$ converges in distribution to the following expression:

$$T^{-\frac{1}{2}} t_{\hat{\gamma}_z} \xrightarrow{d} \frac{\sqrt{3}\left(\int \omega_z - 2\int r\omega_z\right)}{\sqrt{\left(\int \omega_z\right)^2 - 4\left[\int \omega_z^2 - 3\int \omega_z \int r\omega_z + 3\left(\int r\omega_z\right)^2\right]}}$$

where $z = x$ in case [1] and $z = y$ in case [2].

Second step (Residuals Regression) $u_{yt} = \alpha_f + \beta_f \hat{u}_{xt} + \epsilon_t$

Proposition

Let x_t and y_t be generated by DGPs i and j for $i, j = 1, 2, 3,$ and 4 in table 1 and estimate specification (1); build the estimated residuals, $\hat{u}_{yt}$ and $\hat{u}_{yt}$ and estimate specification (2). The orders in probability of $\tau^{w}_{\hat{\beta}_f}$, associated to the regressor $\hat{u}_{xt}$, are:

- $\tau^{r}_{\hat{\beta}_f} = O_p(1)$ *when both variables are independently generated and have a unit root (DGPs 1 and 2),*
- $\hat{\beta}_f \xrightarrow{p} \beta_y$ *and* $\tau^{r}_{\hat{\beta}_f} = O_p\left(\sqrt{T}\right)$ *when both variables are cointegrated (DGPs 1 and 2 for x_t and 4 for y_t).*

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Proposition

Let x_t and y_t be generated by DGPs i and j for $i, j = 1, 2, 3,$ and 4 in table 1 and estimate specification (1); build the estimated residuals, $\hat{u}_{yt}$ and $\hat{u}_{yt}$ and estimate specification (2). The orders in probability of $\tau^{w}_{\hat{\beta}_f}$, associated to the regressor $\hat{u}_{xt}$, are:

- $\tau^{n}_{\hat{\beta}_f} = O_p(1)$ *when both variables are independently generated as TS processes (DGP3) or, both variables are independently generated and one of them is TS (DGP 3) whilst the other has a unit root (DGPs 1 and 2),*
- $\hat{\beta}_f \xrightarrow{p} \beta_y$ *and* $\tau^{n}_{\hat{\beta}_f} = O_p\left(\sqrt{T}\right)$ *when both variables co-trend (DGP 3 for x_t and DGP 4 for y_t).*

http://dl.getdropbox.com/u/1307356/Spurious/spur.pdf

Level	$TS - TS$	$I(1) - TS$	$I(1) - I(1)$
0.010	± 2.576	± 8.295	± 1.292
0.025	± 2.241	± 6.344	± 1.093
0.050	± 1.960	± 4.904	± 0.936
0.100	± 1.645	± 3.553	± 0.764
0.200	± 1.282	± 2.300	± 0.580
0.300	± 1.036	± 1.642	± 0.463
0.400	± 0.842	± 1.204	± 0.373
0.500	± 0.675	± 0.886	± 0.298
0.600	± 0.524	± 0.639	± 0.231
0.700	± 0.385	± 0.441	± 0.169
0.800	± 0.253	± 0.273	± 0.111
0.900	± 0.126	± 0.130	± 0.055
0.950	± 0.063	± 0.064	± 0.027
0.975	± 0.031	± 0.031	± 0.014
0.990	± 0.013	± 0.013	± 0.006

Table: Asymptotic critical values for $\tau_{\hat{\beta}_f^w}$ under the null hypothesis of spurious regression.

DISTRIBUTION UNDER THE NULL HYPOTHESIS

Figure: $\tau^{W}_{\hat{\beta}_f}$ Distribution under the Null Hypothesis

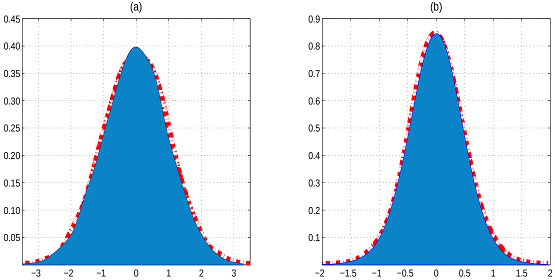

Figure: (a) Combination TS vs TS; (b) Combination $I(1)$ vs $I(1)$. $R = 10,000$. Distribution using simulated data: dash-dotted line. Asimptotic distribution: area.

Rejection rates

Stochastic trend: when the null is true:

Relationship	Parameters					Sample Size				
	μ_x	μ_y	β_y	$\rho_{x,1}$	$\rho_{y,1}$	50	100	200	300	500
Spurious	0.00	0.00	–	0.00	0.00	0.02	0.02	0.02	0.03	0.02
			–	0.50	0.50	0.07	0.04	0.04	0.03	0.03
	0.30	0.50	–	0.00	0.00	0.02	0.02	0.03	0.02	0.02
			–	0.50	0.50	0.07	0.05	0.04	0.04	0.03
		1.25	–	0.00	0.00	0.02	0.02	0.02	0.02	0.02
			–	0.50	0.50	0.07	0.05	0.04	0.03	0.03
		1.50	–	0.00	0.00	0.02	0.03	0.02	0.03	0.02
			–	0.50	0.50	0.07	0.05	0.04	0.03	0.03
	0.50	0.30	–	0.00	0.00	0.02	0.03	0.02	0.02	0.02
			–	0.50	0.50	0.07	0.05	0.04	0.03	0.03
		1.25	–	0.00	0.00	0.02	0.02	0.02	0.02	0.02
			–	0.50	0.50	0.07	0.04	0.04	0.03	0.03
		1.50	–	0.00	0.00	0.02	0.02	0.02	0.02	0.02
			–	0.50	0.50	0.07	0.05	0.04	0.03	0.03
	1.25	0.30	–	0.00	0.00	0.02	0.03	0.02	0.02	0.03
			–	0.50	0.50	0.07	0.05	0.03	0.03	0.03
		0.50	–	0.00	0.00	0.02	0.02	0.03	0.03	0.02
			–	0.50	0.50	0.07	0.04	0.03	0.03	0.03
		1.50	–	0.00	0.00	0.02	0.02	0.02	0.03	0.02
			–	0.50	0.50	0.06	0.05	0.03	0.03	0.03
	1.50	0.30	–	0.00	0.00	0.02	0.02	0.03	0.03	0.02
			–	0.50	0.50	0.07	0.05	0.03	0.03	0.03
		0.50	–	0.00	0.00	0.02	0.02	0.02	0.02	0.02
			–	0.50	0.50	0.07	0.04	0.03	0.03	0.03
		1.25	–	0.00	0.00	0.03	0.02	0.03	0.02	0.03
			–	0.50	0.50	0.07	0.04	0.03	0.03	0.03

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Relationship	Parameters					Sample Size				
	μ_x	μ_y	β_y	$\rho_{x,1}$	$\rho_{y,1}$	50	100	200	300	500
Cointegrated	0.00	0.30	0.30	0.00	0.00	0.01	0.10	0.41	0.65	0.91
				0.50	0.50	0.03	0.62	0.92	0.99	1.00
	0.30	0.50	0.30	0.00	0.00	0.02	0.10	0.40	0.65	0.91
				0.50	0.50	0.28	0.61	0.93	0.99	1.00
			0.50	0.00	0.00	0.24	0.62	0.94	0.99	1.00
				0.50	0.50	0.74	0.96	1.00	1.00	1.00
			1.25	0.00	0.00	0.99	1.00	1.00	1.00	1.00
				0.50	0.50	1.00	1.00	1.00	1.00	1.00
			1.50	0.00	0.00	1.00	1.00	1.00	1.00	1.00
				0.50	0.50	1.00	1.00	1.00	1.00	1.00
			1.75	0.00	0.00	1.00	1.00	1.00	1.00	1.00
				0.50	0.50	1.00	1.00	1.00	1.00	1.00
			2.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00
				0.50	0.50	1.00	1.00	1.00	1.00	1.00

Rejection rates

deterministic trend: when the null is true

Relationship	β_x	β_y	$\rho_{x,1}$	$\rho_{y,1}$	50	100	200	300	500
Spurious	0.30	0.50	0.00	0.00	0.03	0.03	0.03	0.02	0.03
			0.50	0.50	0.09	0.08	0.08	0.08	0.08
		1.25	0.00	0.00	0.03	0.03	0.03	0.03	0.03
			0.50	0.50	0.09	0.08	0.09	0.09	0.08
		1.50	0.00	0.00	0.03	0.03	0.03	0.03	0.03
			0.50	0.50	0.08	0.08	0.09	0.08	0.09
	0.50	0.30	0.00	0.00	0.03	0.03	0.03	0.03	0.03
			0.50	0.50	0.09	0.09	0.09	0.08	0.09
		1.25	0.00	0.00	0.03	0.03	0.03	0.03	0.03
			0.50	0.50	0.09	0.09	0.08	0.08	0.08
		1.50	0.00	0.00	0.03	0.03	0.02	0.03	0.02
			0.50	0.50	0.08	0.09	0.09	0.09	0.08
	1.25	0.30	0.00	0.00	0.03	0.03	0.03	0.03	0.03
			0.50	0.50	0.09	0.09	0.08	0.09	0.08
		0.50	0.00	0.00	0.03	0.03	0.03	0.03	0.03
			0.50	0.50	0.09	0.09	0.08	0.08	0.08
		1.50	0.00	0.00	0.03	0.03	0.03	0.03	0.03
			0.50	0.50	0.09	0.08	0.08	0.08	0.08
	1.50	0.30	0.00	0.00	0.03	0.03	0.02	0.03	0.03
			0.50	0.50	0.09	0.09	0.08	0.08	0.08
		0.50	0.00	0.00	0.03	0.03	0.03	0.03	0.03
			0.50	0.50	0.08	0.08	0.08	0.08	0.09
		1.25	0.00	0.00	0.03	0.03	0.03	0.02	0.03
			0.50	0.50	0.09	0.09	0.08	0.09	0.08

Rejection rates

Stochastic trend: when the null is false

Relationship	Parameters				Sample Size				
	β_x	β_y	$\rho_{x,1}$	$\rho_{y,1}$	50	100	200	300	500
Co-trended	0.30	0.30	0.00	0.00	0.45	0.76	0.97	1.00	1.00
			0.50	0.50	0.47	0.72	0.94	0.98	1.00
		0.50	0.00	0.00	0.87	1.00	1.00	1.00	1.00
			0.50	0.50	0.81	0.98	1.00	1.00	1.00
		1.25	0.00	0.00	1.00	1.00	1.00	1.00	1.00
			0.50	0.50	1.00	1.00	1.00	1.00	1.00
		1.50	0.00	0.00	1.00	1.00	1.00	1.00	1.00
			0.50	0.50	1.00	1.00	1.00	1.00	1.00
		1.75	0.00	0.00	1.00	1.00	1.00	1.00	1.00
			0.50	0.50	1.00	1.00	1.00	1.00	1.00
		2.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00
			0.50	0.50	1.00	1.00	1.00	1.00	1.00

Power-level tradeoff curve

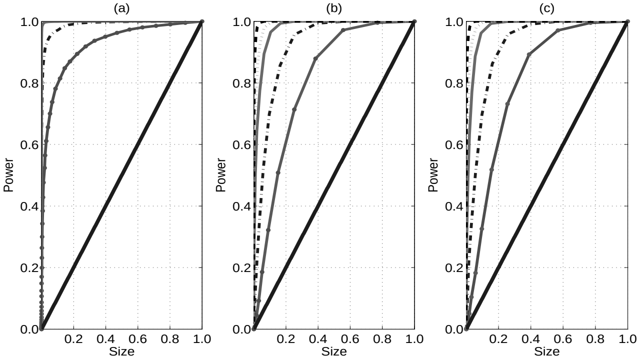

Figure: Size-Power tradeoff Curve: (a) *TS* vs *TS*; (b) $I(1)$ with drift vs $I(1)$ with drift; (c) driftless $I(1)$ vs driftless $I(1)$; Sample sizes: $T = 50$ (round marker), 100 (dash-dotted line), 200 (solid line), 300 (dotted line), 500 (dashed line).

So far, we know: i ii iii iv

Spurious Regression occurs under a wide variety of Data Generating Processes: (broken-)trend stationary, stationary, long memory, unit root,...

So far, we know: i ii iii iv

There are many tests that could be used to “prevent” spurious regression: cointegration (extremely popular), co-trending (not that popular)

So far, we know: i ii iii iv

We introduce a test circumscribed to processes with neither structural breaks, nor long memory. The test, assisted by a pre-testing strategy, distinguishes spurious regression from cointegration/co-trending.

So far, we know: i ii iii iv

Asymptotic critical values have been computed; finite sample evidence shows that the test may be useful for practical purposes.

And yet, we should: i ii iii

Compare our proposal with other tests.

And yet, we should: i ii iii

Extend our results for DGPs with structural breaks.

And yet, we should: i ii iii

Employ our test in an empirical application.

The paper can be downloaded at:

http://www.ventosa-santaularia.com

Thank you

How to obtain the asymptotics

We present a case where the simplest DGP–Driftless UR–is used.

http://dl.getdropbox.com/u/1307356/Spurious/spur.pdf

Example: Remember that $x_t = \mu_x + u_{xt}$

- The auxiliary regression is: $x_t = \gamma + \tau \cdot t + u_t$
- or... $Y = X\delta + U$
- The estimator is $\hat{\delta} = (X'X)^{-1} X'Y$

We require–among other sums:

- $\sum x_t = \mu_x T + \underbrace{\sum u_{xt}}_{O_p\left(T^{\frac{1}{2}}\right)} = O_p\left(T\right)$
- $\sum x_t \cdot t = \mu_x \sum t + \underbrace{\sum u_{xt} t}_{O_p\left(T^{\frac{3}{2}}\right)} = O_p\left(T^2\right)$

For all the calculations we need:

$\sum_{t=1}^{T} x_t = O_p(T)$	$\sum_{t=1}^{T} (x_t)^2 = O_p(T)$
$\sum_{t=1}^{T} x_t \cdot t = O_p(T^2)$	$\sum_{t=1}^{T} t^2 = O(T^3)$

Finally

- We "fill" all the relevant matrices, that is: $X'X$ and $X'Y$
- The asymptotics calculations are software-assisted.

▸ Results