Estimación de Reservas Mediante un Método de Pronóstico Bayesiano para Series Cortas con Estacionalidad

Enrique de Alba

Seminario Aleatorio

$25_{años}$ | $\ddot{a}ctuaría$

Claims reserving methods:

- Deterministic.
- Stochastic.

Stochastic models are too complex and tend to be overparameterized.

Model historical observations very accurately, but do not necessarily perform well when predicting future claims.

ITAM

Bayesian methods developed for forecasting very short series with stable (seasonal) patterns. At any given time $t > t_0$

, $Z(t)$ represents the accumulated value of the variable Z over the interval $(t_0, t]$. Series of interest is $\{X_i\}$, $i = 1 \ldots n$;

X_i to the accumulated value of Z over the *i*-th year.

Series $\{Y_i^{\alpha}\}$, a partial accumulation process is also available. Y_i^{α} stands for the accumulated value of Z over the first month $(\alpha = 1/12)$

Given $Y_{n+1}^{\alpha} = y$, X_{n+1} is obtained as a linear transformation of the ratio

$$W_{n+1}^{\alpha} = \frac{(X_{n+1} - y)}{y}$$

The predictive distribution of W_{n+1}^{α} is obtained as a log – Student distribution and

$$X_{n+1}^{(q)} = y + y\left[\prod_{i=1}^{n} W_i^{\alpha}\right]^{1/n}\left[\exp(t_{n+1}^{(q)})\right]^{\tilde{\sigma}(1+\frac{1}{n})^{1/2}}$$

ITAM

Estimator for the total of $n+1$ year given accumulation up to α

$$\hat{X}_{n+1} = y \left(1 + \prod_{i=1}^{n} W_i^{\alpha} \right)^{1/n}$$

ITAM

Expenses
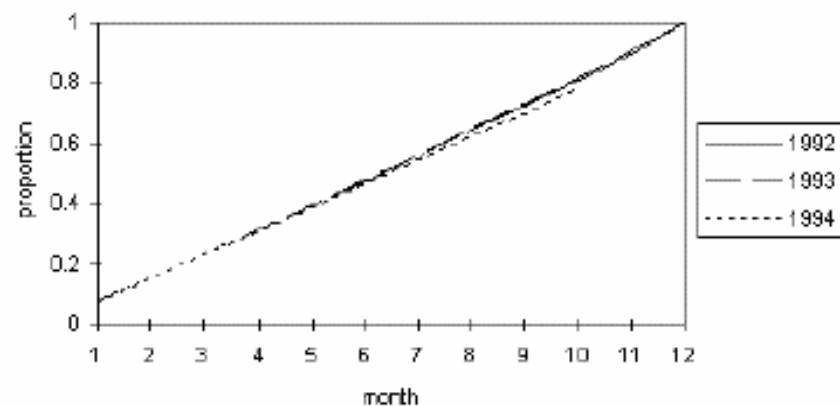
Electricity

Mendoza, M. and de Alba, E. (2006), "Forecasting an Accumulated Series Based on Partial Accumulation II: A New Bayesian Method for Short Series with Seasonal Patterns", *International Journal of Forecasting* 4, Oct-Dec, 781-798.

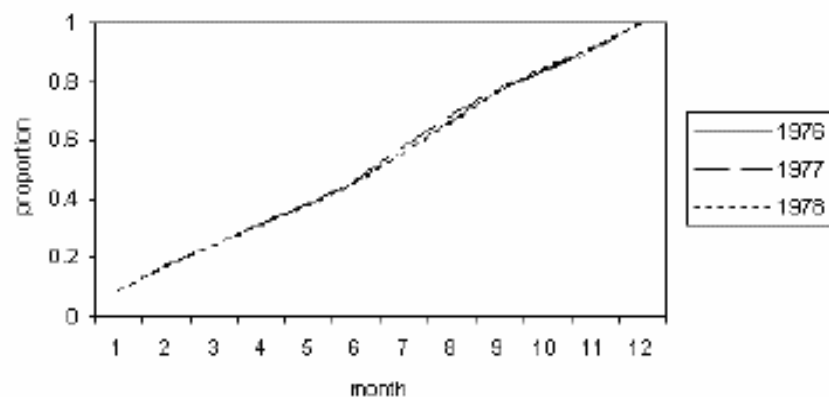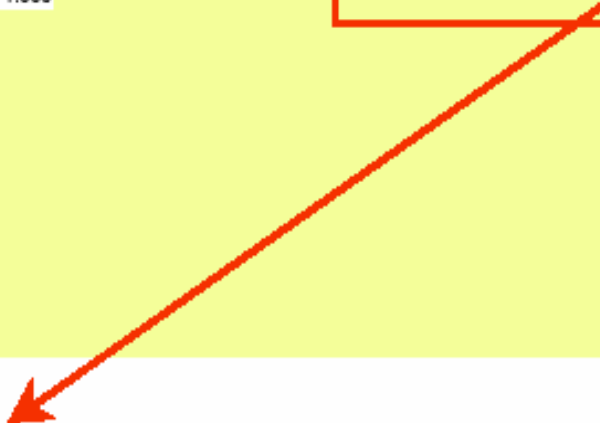|		
nov-06	10.800	86.502
dic-06	9.501	-

jul-05	ago-05	sep-05	oct-05	nov-05	dic-05	ene-06	feb-06	****	****	*******	jun-06	jul-06	****	sep-06	oct-06	nov-06	dic-06
0.007	-	0.006	0.002	0.002	0.009	0.008	0.000	-	-	0.045	0.000	0.000	-	-	0.001	0.000	0.000
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.138	-
-	-	-	-	0.518	-	0.029	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	0.011	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.387	-	-	-	0.000	-	-	0.002	-	-	0.189	0.000	0.004	-	-	-	-	-
-	-	-	0.021	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.180	-	0.054	-	-0.002	-	0.019	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	0.065	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	0.809	-	0.001	-	-	-	-	-	-	-	-	-	-	-	-
-	0.018	-	0.001	0.001	0.001	0.000	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	0.001	0.000	-	-	-	-	-	-	-	-	-	-	-	-
0.026	-	-	0.018	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.002	0.002	0.167	0.001	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.007	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.001	0.000	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.002	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2001-1	264.279	322.762	59.799	18.653	4.476	4.781	2.796	2.356	1.263	0.799	0.431	0.972	0.274	0.507	0.218	0.267	0.314	0.072	0.340	0.073	0.148	0.045	0.000	0.032
2001-2	274.292	358.070	71.054	11.562	9.283	5.850	3.937	1.317	2.316	1.799	0.839	0.538	0.615	0.377	0.561	-0.048	-0.039	0.066	0.307	0.000	0.000	0.002	0.110	
2001-3	289.229	385.881	39.257	22.241	10.045	6.446	2.955	4.735	3.939	3.066	0.868	1.010	1.091	1.394	0.163	0.941	0.090	0.049	0.100	0.075	0.016	0.000		
2001-4	304.909	337.193	85.376	27.148	15.928	4.631	3.529	3.929	2.081	0.863	1.256	0.676	0.402	1.093	0.602	0.548	0.292	0.098	0.202	0.025	0.003			
2002-1	239.667	401.053	78.387	26.534	7.960	6.576	6.427	3.557	1.469	2.083	1.278	0.404	0.292	0.567	1.144	0.120	0.012	0.009	0.149	0.169				
2002-2	304.929	416.627	86.106	16.674	12.487	9.110	5.039	2.573	2.339	2.409	0.898	0.845	0.442	0.745	0.428	0.343	0.500	0.066	0.112					
2002-3	327.187	490.316	62.423	31.480	15.222	8.746	4.596	5.045	2.705	2.052	1.334	1.163	0.565	0.849	0.895	0.315	0.187	0.071						
2002-4	381.025	410.039	108.427	36.198	18.665	5.671	5.308	3.129	3.347	2.037	2.500	0.786	1.256	0.388	1.019	0.348	0.371							
2003-1	295.942	430.232	79.464	28.152	10.828	6.734	3.203	2.692	2.072	1.200	1.735	0.870	0.439	0.182	0.968	0.183								
2003-2	334.195	457.061	102.172	17.906	12.140	6.500	5.879	1.770	2.493	0.713	0.904	0.945	0.306	0.470	0.254									
2003-3	338.088	519.565	60.393	28.093	12.073	8.982	3.825	4.085	2.150	1.260	0.730	0.652	0.623	0.560										
2003-4	357.203	438.350	101.980	34.557	16.332	6.041	4.982	3.655	3.022	0.795	0.435	0.821	0.306											
2004-1	334.626	483.412	98.116	34.635	9.151	6.249	4.819	3.457	1.672	2.556	0.979	1.024												
2004-2	351.198	512.403	120.461	19.427	16.356	8.253	6.837	2.140	3.766	0.940	1.469													
2004-3	356.257	610.121	89.973	48.184	22.753	13.724	6.225	6.159	2.922	1.958														
2004-4	387.015	587.841	182.651	52.200	26.072	9.890	7.173	5.892	2.597															
2005-1	290.718	649.937	150.065	51.467	13.767	7.947	6.143	4.545																
2005-2	372.955	708.232	172.666	28.833	16.828	11.397	6.643																	
2005-3	434.750	801.084	108.409	48.112	23.367	10.591																		
2005-4	485.735	748.773	206.781	64.115	24.019																			
2006-1	374.480	789.839	167.892	53.089																				
2006-2	445.191	775.337	193.892																					
2006-3	370.377	789.096																						
2006-4	426.115																							

1.000406	1.000263	1.000077	1.000261	1.000091	1.000056	1.000021	1.000077	1.000046	1.001300

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2001-1	264.279	322.762	59.799	18.653	4.476	4.781	2.796	2.356	1.263	0.799	0.431	0.972	0.274	0.507	0.218	0.267
2001-2	274.292	358.070	71.054	11.562	9.283	5.850	3.937	1.317	2.316	1.799	0.839	0.538	0.615	0.377	0.561	-0.048
2001-3	289.229	385.881	39.257	22.241	10.045	6.446	2.955	4.735	3.939	3.066	0.868	1.010	1.091	1.394	0.163	0.941
2001-4	304.909	337.193	85.376	27.148	15.928	4.631	3.529	3.929	2.081	0.863	1.256	0.676	0.402	1.093	0.602	0.548
2002-1	239.667	401.053	78.387	26.534	7.960	6.576	6.427	3.557	1.469	2.083	1.278	0.404	0.292	0.567	1.144	0.120
2002-2	304.929	416.627	86.106	16.674	12.487	9.110	5.039	2.573	2.339	2.409	0.896	0.845	0.442	0.745	0.428	0.343
2002-3	327.187	490.316	62.423	31.480	15.222	8.746	4.596	5.045	2.705	2.052	1.334	1.163	0.565	0.849	0.895	0.315
2002-4	381.025	410.039	108.427	36.198	18.665	5.671	5.308	3.129	3.347	2.037	2.500	0.788	1.256	0.388	1.019	0.348
2003-1	295.942	430.232	79.464	28.152	10.828	6.734	3.203	2.692	2.072	1.200	1.735	0.870	0.439	0.182	0.966	0.183
2003-2	334.195	457.061	102.172	17.906	12.140	6.500	5.879	1.770	2.493	0.713	0.904	0.945	0.306	0.470	0.254	
2003-3	338.088	519.565	60.393	28.093	12.073	8.982	3.825	4.085	2.150	1.260	0.730	0.652	0.623	0.560		
2003-4	357.203	438.350	101.980	34.557	16.332	6.041	4.982	3.655	3.022	0.795	0.435	0.821	0.306			
2004-1	334.626	483.412	98.116	34.635	9.151	6.249	4.819	3.457	1.672	2.556	0.979	1.024				
2004-2	351.198	512.403	120.461	19.427	16.356	8.253	6.837	2.140	3.766	0.940	1.469					
2004-3	356.257	610.121	89.973	48.184	22.753	13.724	6.225	6.159	2.922	1.958						
2004-4	387.015	587.841	182.651	52.200	26.072	9.890	7.173	5.892	2.597							
2005-1	290.718	649.937	150.065	51.467	13.767	7.947	6.143	4.545								
2005-2	372.955	708.232	172.666	28.833	16.828	11.397	6.643									
2005-3	434.750	801.084	108.409	48.112	23.367	10.591										
2005-4	485.735	748.773	206.781	64.115	24.019											
2006-1	374.480	789.839	167.892	53.089												
2006-2	445.191	775.337	193.892													
2006-3	370.377	789.096														
2006-4	426.115															

Compare several methods.

- Chain ladder.

- Log – linear model with a trend.

- Bayesian Log linear model w/trend (WinBUGS).

- Bayesian Forecasting.

- De Jong (Kalman Filter).

ITAM

Development year (j)/Accident year (i)	1	2	3	4	5	6	7	8	
2	274.29	358.07	71.05	11.56	9.28	5.85	3.94	1.32	
3	289.23	385.88	39.26	22.24	10.04	6.45	2.95	4.74	4.74
4	304.91	337.19	85.38	27.15	15.93	4.63	3.53	3.93	7.46
5	239.67	401.05	78.39	26.53	7.96	6.58	6.43	3.56	16.56
6	304.93	416.63	86.11	16.67	12.49	9.11	5.04	2.57	29.21
7	327.19	490.32	62.42	31.48	15.22	8.75	4.60	5.04	65.09
8	381.03	410.04	108.43	36.20	18.66	5.67	5.31	3.13	177.40
9	295.94	430.23	79.46	28.15	10.83	6.73	3.20	2.69	561.31
							Reserve		861.76

Comparison among different methods						
Accident year	Regression	Bayesian	Winbugs	Chain Ladder	De Jong (H)	De Jong (D)
3	255.43%	150.70%	245.80%	248.96%	71.34%	71.34%
4	28.78%	29.78%	46.93%	49.16%	32.84%	32.84%
5	51.65%	45.75%	56.84%	56.40%	35.91%	35.91%
6	29.08%	42.60%	36.16%	22.62%	18.46%	18.47%
7	42.46%	53.52%	46.25%	27.36%	21.31%	21.33%
8	37.61%	39.71%	42.74%	34.79%	25.14%	25.24%
9	11.67%	16.65%	2.36%	11.21%	15.02%	3.18%
Average	65.24%	54.10%	68.15%	64.36%	31.43%	29.76%

Comparison among different methods						
Accident year	Regression	Bayesian	Winbugs	Chain Ladder	De Jong (H)	De Jong (D)
4	30.12%	**50.47%**	24.92%	**19.45%**	24.15%	24.15%
5	19.15%	**53.22%**	19.70%	**24.70%**	19.85%	19.85%
6	12.47%	**17.28%**	15.58%	**8.67%**	7.92%	7.92%
7	12.63%	**20.53%**	25.93%	**9.00%**	8.28%	8.30%
8	5.81%	**41.14%**	27.74%	**14.17%**	12.26%	12.26%
9	2.39%	**25.54%**	8.85%	**0.63%**	1.00%	0.86%
10	2.62%	**9.07%**	0.72%	**2.30%**	0.88%	2.88%
Average	**12.17%**	**31.03%**	**17.64%**	**11.28%**	**10.62%**	**10.89%**

Comparison among different methods						
Accident year	Regression	Bayesian	Winbugs	Chain Ladder	De Jong (H)	De Jong (D)
5	10.49%	**17.45%**	14.18%	**8.02%**	8.72%	8.72%
6	18.63%	**9.22%**	21.52%	**22.18%**	28.72%	28.71%
7	3.62%	**15.13%**	0.07%	**2.10%**	1.92%	1.91%
8	9.96%	**0.33%**	6.16%	**4.13%**	4.08%	4.07%
9	7.97%	**8.74%**	15.74%	**11.03%**	12.21%	12.20%
10	4.21%	**10.99%**	1.42%	**5.42%**	6.24%	6.16%
11	4.53%	**17.89%**	4.18%	**4.47%**	2.93%	2.07%
Average	**8.49%**	**11.39%**	**9.04%**	**8.19%**	**9.26%**	**9.12%**

ITAM

Comparison among different methods						
Accident year	Regression	Bayesian	Winbugs	Chain Ladder	De Jong (H)	De Jong (D)
6	32.10%	**37.51%**	32.35%	**34.85%**	53.48%	53.48%
7	13.52%	**8.32%**	5.25%	**12.75%**	15.27%	15.26%
8	41.72%	**20.38%**	32.82%	**31.67%**	46.65%	46.65%
9	27.15%	**19.48%**	31.39%	**25.73%**	33.78%	33.77%
10	34.19%	**32.58%**	36.60%	**32.21%**	46.79%	46.77%
11	29.22%	**34.88%**	30.73%	**30.33%**	43.60%	43.50%
12	9.96%	**8.92%**	7.52%	**9.78%**	12.17%	10.27%
Average	**26.84%**	**23.15%**	**25.24%**	**25.33%**	**35.96%**	**35.67%**

Comparison among different methods						
Accident year	Regression	Bayesian	Winbugs	Chain Ladder	De Jong (H)	De Jong (D)
7	79.64%	**43.29%**	79.80%	**77.51%**	43.67%	43.67%
8	1.35%	**19.15%**	4.98%	**4.03%**	3.58%	3.58%
9	10.31%	**28.65%**	15.53%	**12.96%**	15.74%	15.72%
10	6.93%	**15.49%**	9.19%	**14.60%**	17.11%	17.11%
11	11.59%	**2.14%**	4.85%	**4.72%**	4.82%	4.79%
12	21.40%	**9.56%**	20.69%	**18.19%**	14.95%	15.04%
13	8.79%	**4.18%**	2.77%	**7.95%**	6.66%	12.73%
Average	**20.00%**	17.49%	19.69%	19.99%	15.22%	16.09%

ITAM

Comparison among different methods						
Accident year	Regression	Bayesian	Winbugs	Chain Ladder	De Jong (H)	De Jong (D)
8	42.90%	**19.53%**	42.97%	**39.60%**	49.38%	49.38%
9	32.59%	**28.09%**	42.00%	**34.80%**	42.09%	42.09%
10	6.15%	**23.05%**	16.06%	**17.01%**	18.71%	18.70%
11	0.25%	**15.31%**	4.03%	**11.48%**	11.98%	11.97%
12	4.57%	**12.69%**	15.25%	**7.50%**	7.23%	7.24%
13	7.66%	**13.21%**	7.28%	**4.49%**	3.95%	4.06%
14	9.44%	**5.96%**	6.57%	**8.74%**	7.65%	6.77%
Average	**14.79%**	**16.83%**	**19.17%**	**17.66%**	**20.14%**	**20.03%**

Comparison among different methods						
Accident year	Regression	Bayesian	Winbugs	Chain Ladder	De Jong (H)	De Jong (D)
9	8.81%	**24.48%**	12.86%	**2.82%**	2.86%	2.86%
10	20.57%	**13.58%**	8.55%	**3.38%**	3.99%	4.00%
11	38.40%	**2.93%**	25.03%	**17.88%**	16.49%	16.49%
12	6.59%	**7.43%**	13.71%	**10.96%**	10.61%	10.61%
13	1.10%	**15.49%**	6.62%	**0.23%**	0.20%	0.18%
14	10.65%	**7.40%**	18.46%	**7.81%**	7.21%	7.30%
15	25.12%	**31.14%**	19.43%	**24.41%**	21.14%	1.07%
Average	**15.89%**	**14.63%**	**14.95%**	**9.64%**	**8.93%**	**6.07%**

ITAM

Comparison among different methods						
Accident year	Regression	Bayesian	Winbugs	Chain Ladder	De Jong (H)	De Jong (D)
10	40.22%	**40.81%**	35.96%	**39.96%**	49.94%	49.94%
11	3.10%	**4.83%**	3.33%	**1.05%**	0.37%	0.36%
12	17.82%	**4.10%**	13.55%	**6.09%**	5.82%	5.81%
13	25.34%	**20.67%**	26.13%	**19.40%**	21.13%	21.14%
14	20.43%	**23.13%**	21.61%	**16.14%**	17.44%	17.43%
15	6.42%	**3.81%**	3.47%	**3.14%**	3.03%	3.13%
16	18.24%	**31.72%**	17.92%	**17.38%**	15.97%	20.49%
Average	**18.80%**	**18.44%**	**17.43%**	**14.74%**	**16.24%**	**16.90%**

Comparison among different methods						
Accident year	Regression	Bayesian	Winbugs	Chain Ladder	De Jong (H)	De Jong (D)
11	132.38%	**72.37%**	110.59%	**122.45%**	75.95%	75.95%
12	17.89%	**19.90%**	19.15%	**28.98%**	24.98%	24.99%
13	3.98%	**0.03%**	3.51%	**3.11%**	2.97%	2.98%
14	27.26%	**17.49%**	29.23%	**18.87%**	17.12%	17.12%
15	72.83%	**63.20%**	80.96%	**57.89%**	44.60%	44.63%
16	69.20%	**59.36%**	78.61%	**64.27%**	48.04%	48.15%
17	61.30%	**64.29%**	23.45%	**59.57%**	45.79%	22.15%
Average	**54.98%**	**42.38%**	**49.36%**	**50.74%**	**37.06%**	**33.71%**

ITAM

	Regression	Bayesian	WingBUGS	Chain ladder	De Jong (H)	De Jong(D)
Triangle 2	65.24%	54.10%	68.15%	64.36%	31.43%	29.76%
Triangle 3	12.17%	31.03%	17.64%	11.28%	10.62%	10.89%
Triangle 4	8.49%	11.39%	9.04%	8.19%	9.26%	9.12%
Triangle 5	26.84%	23.15%	25.24%	25.33%	35.96%	35.67%
Triangle 6	20.00%	17.49%	19.69%	19.99%	15.22%	16.09%
Triangle 7	14.79%	16.83%	19.17%	17.66%	20.14%	20.03%
Triangle 8	15.89%	14.63%	14.95%	9.64%	8.93%	6.07%
Triangle 9	18.80%	18.44%	17.43%	14.74%	16.24%	16.90%
Triangle 10	54.98%	42.38%	49.36%	50.74%	37.06%	33.71%
Average	26.35%	25.50%	26.74%	24.66%	18.48%	18.07%

ITAM

	Regression Model								
Accident year	Triangle 2	Triangle 3	Triangle 4	Triangle 5	Triangle 6	Triangle 7	Triangle 8	Triangle 9	Triangle 10
2	255.43%	30.12%	10.49%	32.10%	79.64%	42.90%	8.81%	40.22%	132.38%
3	28.78%	19.15%	18.63%	13.52%	1.35%	32.59%	20.57%	3.10%	17.89%
4	51.65%	12.47%	3.62%	41.72%	10.31%	6.15%	38.40%	17.82%	3.98%
5	29.08%	12.63%	9.96%	27.15%	6.93%	0.25%	6.59%	25.34%	27.26%
6	42.46%	5.81%	7.97%	34.19%	11.59%	4.57%	1.10%	20.43%	72.83%
7	37.61%	2.39%	4.21%	29.22%	21.40%	7.66%	10.65%	6.42%	69.20%
8	11.67%	2.62%	4.53%	9.96%	8.79%	9.44%	25.12%	18.24%	61.30%
Average	65.24%	12.17%	8.49%	26.84%	20.00%	14.79%	15.89%	18.80%	54.98%

	Bayesian								
Accident year	Triangle 2	Triangle 3	Triangle 4	Triangle 5	Triangle 6	Triangle 7	Triangle 8	Triangle 9	Triangle 10
2	150.70%	50.47%	17.45%	37.51%	43.29%	19.53%	24.48%	40.81%	72.37%
3	29.78%	53.22%	9.22%	8.32%	19.15%	28.09%	13.58%	4.83%	19.90%
4	45.75%	17.28%	15.13%	20.38%	28.65%	23.05%	2.93%	4.10%	0.03%
5	42.60%	20.53%	0.33%	19.48%	15.49%	15.31%	7.43%	20.67%	17.49%
6	53.52%	41.14%	8.74%	32.58%	2.14%	12.69%	15.49%	23.13%	63.20%
7	39.71%	25.54%	10.99%	34.88%	9.56%	13.21%	7.40%	3.81%	59.36%
8	16.65%	9.07%	17.89%	8.92%	4.18%	5.96%	31.14%	31.72%	64.29%
Average	54.10%	31.03%	11.39%	23.15%	17.49%	16.83%	14.63%	18.44%	42.38%

WinBUGS

Accident year	Triangle 2	Triangle 3	Triangle 4	Triangle 5	Triangle 6	Triangle 7	Triangle 8	Triangle 9	Triangle 10
2	245.80%	24.92%	14.18%	32.35%	79.80%	42.97%	12.86%	35.96%	110.59%
3	46.93%	19.70%	21.52%	5.25%	4.98%	42.00%	8.55%	3.33%	19.15%
4	56.84%	15.58%	0.07%	32.82%	15.53%	16.06%	25.03%	13.55%	3.51%
5	36.16%	25.93%	6.16%	31.39%	9.19%	4.03%	13.71%	26.13%	29.23%
6	46.25%	27.74%	15.74%	36.60%	4.85%	15.25%	6.62%	21.61%	80.96%
7	42.74%	8.85%	1.42%	30.73%	20.69%	7.28%	18.46%	3.47%	78.61%
8	2.36%	0.72%	4.18%	7.52%	2.77%	6.57%	19.43%	17.92%	23.45%
Average	68.15%	17.64%	9.04%	25.24%	19.69%	19.17%	14.95%	17.43%	49.36%

Chain Ladder

Accident year	Triangle 2	Triangle 3	Triangle 4	Triangle 5	Triangle 6	Triangle 7	Triangle 8	Triangle 9	Triangle 10
2	248.96%	19.45%	8.02%	34.85%	77.51%	39.60%	2.82%	39.96%	122.45%
3	49.16%	24.70%	22.18%	12.75%	4.03%	34.80%	3.38%	1.05%	28.98%
4	56.40%	8.67%	2.10%	31.67%	12.96%	17.01%	17.88%	6.09%	3.11%
5	22.62%	9.00%	4.13%	25.73%	14.60%	11.48%	10.96%	19.40%	18.87%
6	27.36%	14.17%	11.03%	32.21%	4.72%	7.50%	0.23%	16.14%	57.89%
7	34.79%	0.63%	5.42%	30.33%	18.19%	4.49%	7.81%	3.14%	64.27%
8	11.21%	2.30%	4.47%	9.78%	7.95%	8.74%	24.41%	17.38%	59.57%
Average	64.36%	11.28%	8.19%	25.33%	19.99%	17.66%	9.64%	14.74%	50.74%

ITAM

	De Jong (Hertig)								
Accident year	Triangle 2	Triangle 3	Triangle 4	Triangle 5	Triangle 6	Triangle 7	Triangle 8	Triangle 9	Triangle 10
2	71.34%	24.15%	8.72%	53.48%	43.67%	49.38%	2.86%	49.94%	75.95%
3	32.84%	19.85%	28.72%	15.27%	3.58%	42.09%	3.99%	0.37%	24.98%
4	35.91%	7.92%	1.92%	46.65%	15.74%	18.71%	16.49%	5.82%	2.97%
5	18.46%	8.28%	4.08%	33.78%	17.11%	11.98%	10.61%	21.13%	17.12%
6	21.31%	12.26%	12.21%	46.79%	4.82%	7.23%	0.20%	17.44%	44.60%
7	25.14%	1.00%	6.24%	43.60%	14.95%	3.95%	7.21%	3.03%	48.04%
8	15.02%	0.88%	2.93%	12.17%	6.66%	7.65%	21.14%	15.97%	45.79%
Average	31.43%	10.62%	9.26%	35.96%	15.22%	20.14%	8.93%	16.24%	37.06%

	De Jong (Development)								
Accident year	Triangle 2	Triangle 3	Triangle 4	Triangle 5	Triangle 6	Triangle 7	Triangle 8	Triangle 9	Triangle 10
2	71.34%	24.15%	8.72%	53.48%	43.67%	49.38%	2.86%	49.94%	75.95%
3	32.84%	19.85%	28.71%	15.26%	3.58%	42.09%	4.00%	0.36%	24.99%
4	35.91%	7.92%	1.91%	46.65%	15.72%	18.70%	16.49%	5.81%	2.98%
5	18.47%	8.30%	4.07%	33.77%	17.11%	11.97%	10.61%	21.14%	17.12%
6	21.33%	12.26%	12.20%	46.77%	4.79%	7.24%	0.18%	17.43%	44.63%
7	25.24%	0.86%	6.16%	43.50%	15.04%	4.06%	7.30%	3.13%	48.15%
8	3.18%	2.88%	2.07%	10.27%	12.73%	6.77%	1.07%	20.49%	22.15%
Average	29.76%	10.89%	9.12%	35.67%	16.09%	20.03%	6.07%	16.90%	33.71%

ITam

	1	2	3	4	5	6
1	8489.00	1296.00	924.00	580.00	246.00	126.00
2	12970.00	1796.00	1435.00	859.00	654.00	265.00
3	17522.00	2783.00	1469.00	1023.00	423.00	652.00
4	21754.00	2584.00	1163.00	783.00	887.00	355.00
5	19208.00	2341.00	1220.00	619.00	841.00	703.00
6	19604.00	2469.00	1223.00	1247.00	612.00	
7	21922.00	2311.00	1141.00	1508.00		
8	25038.00	3363.00	2144.00			
9	32532.00	4474.00				
10	39862.00					

Median
445.36
1296.67
2851.58
6366.46
13300.12
24260.17996

Doray (1996)

Histogram of Doray

ITAM
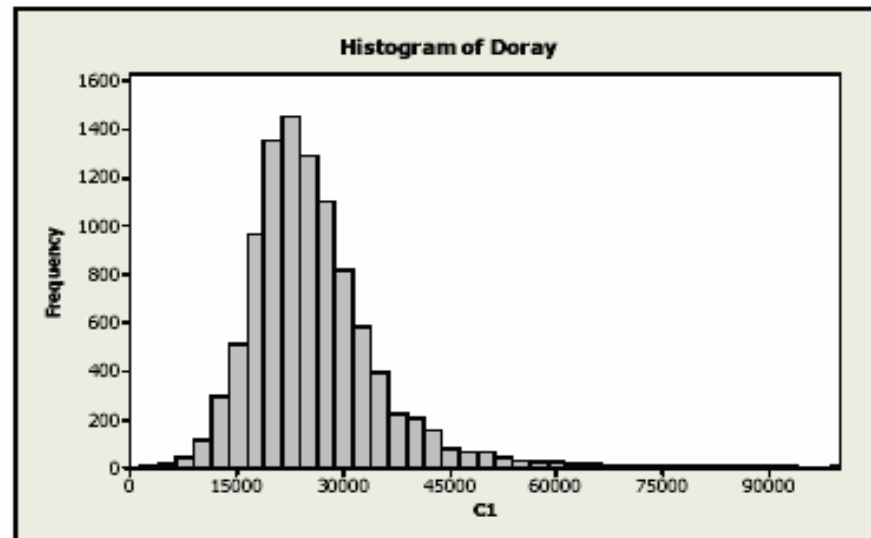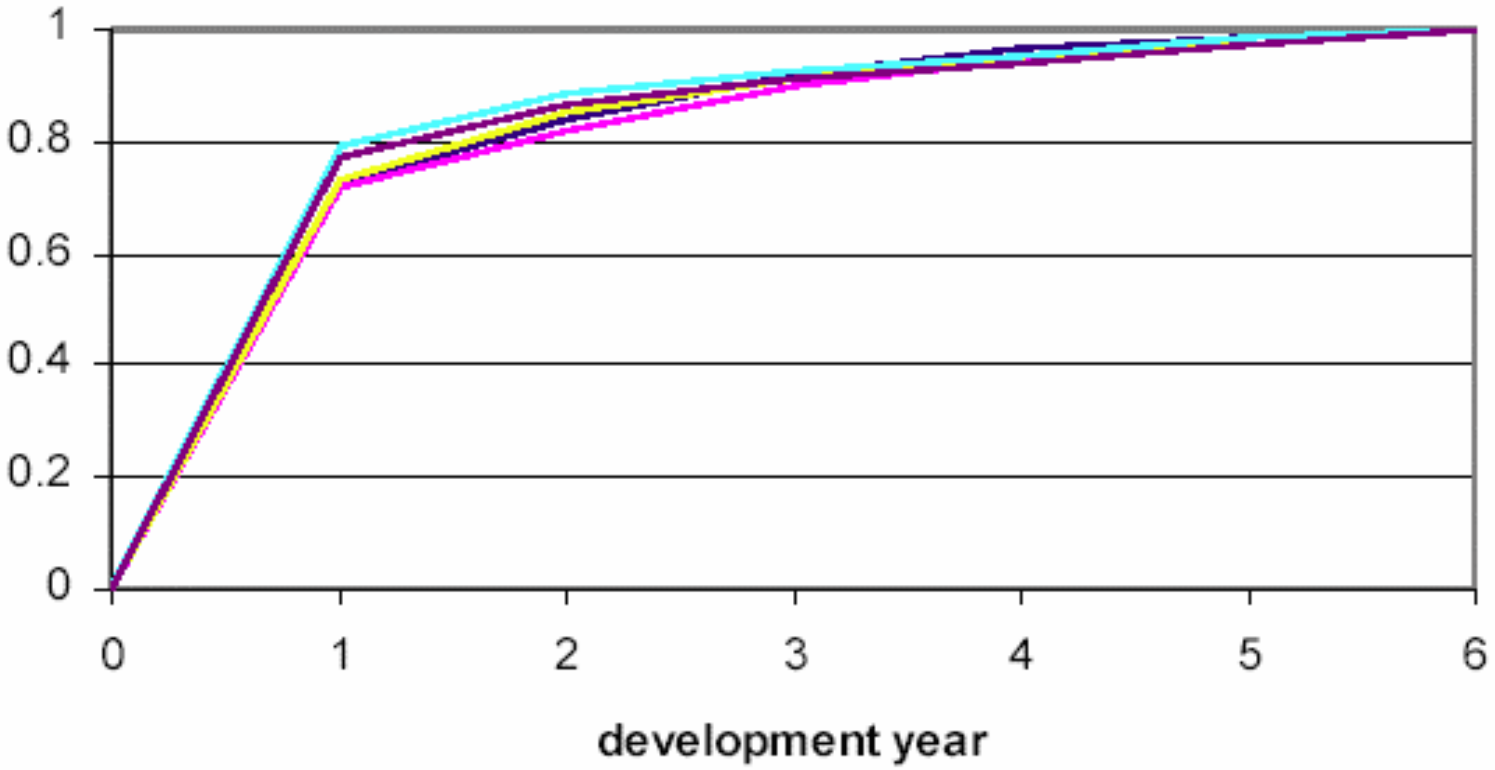
357848	766940	610542	482940	527326	574398	146342	139950	227229		67948
352118	884021	933594	1193289	445745	320996	527804	266172	425046		
290507	1001799	926219	1016654	750816	146923	495992	280405			
310608	1108250	776189	1562400	272482	352053	206286				
443160	693190	991983	769488	504851	470639					
396132	937085	847498	805037	705960						
440832	847631	1131398	1063269							
359480	1061648	1443370								
376686	986608									m9,10
344014										1.018

Partial	Complete
	94,806
362,071	455,504
575,281	666,804
827,630	910,953
1,477,673	1,569,299
2,283,918	2,386,137
3,812,633	3,930,984
3,858,903	3,951,465
4,038,704	4,116,386
17,236,813	18,082,338

Model	Reserves (1,000)
Chain ladder	$18,681
Verrall (no prior info)	19,512
Verrall (prior info)	16,743
Mack	18,085
Renshaw/Christofides	19,512
Bayesian (global)	19,627
Bayesian (exposure)	19,630
Taylor and Ashe	22,301
Bayesian (claims)	25,075

Taylor and Ashe (1983)

Cumulative Percentage
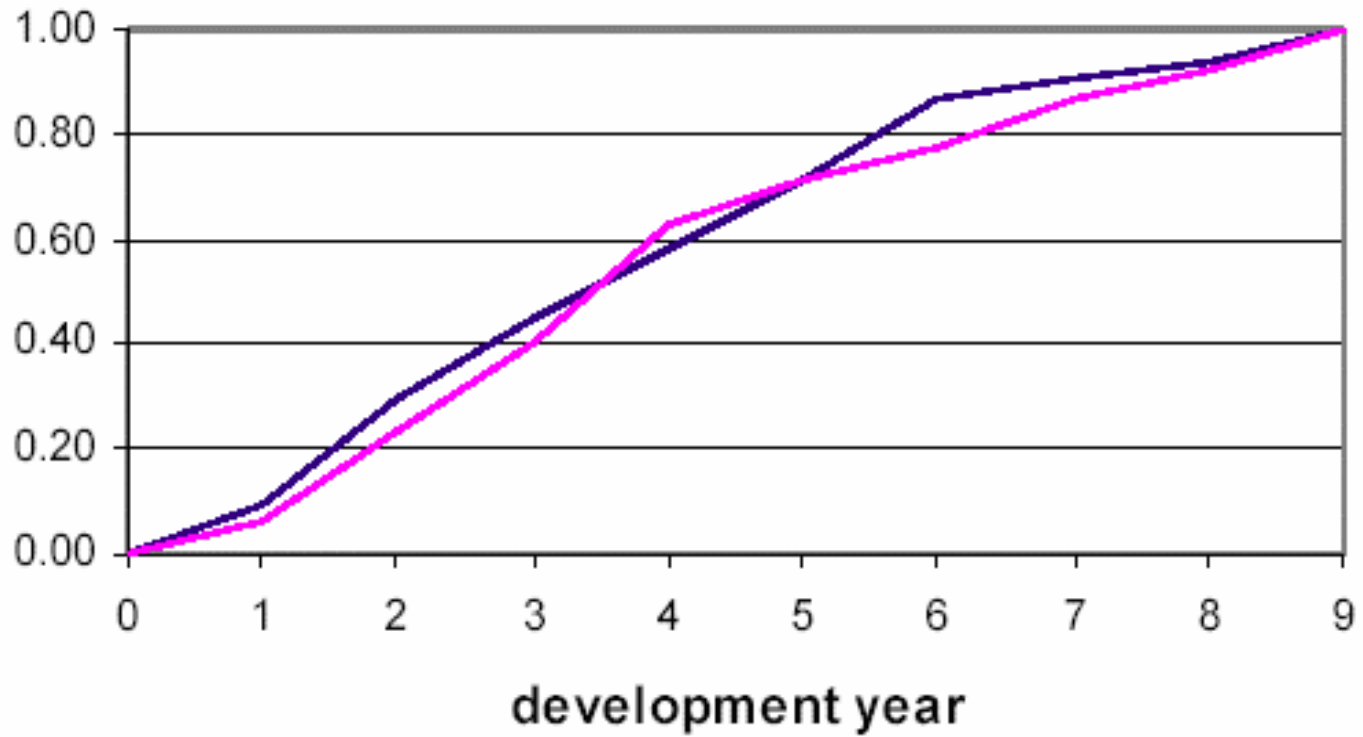
	1	2	3	4	5	6	7	8	9	10
1	5012	3257	2638	898	1734	2642	1828	599	54	172
2	106	4179	1111	5270	3116	1817	-103	673	535	0
3	3410	5582	4881	2268	2594	3479	649	603	0	0
4	5655	5900	4211	5500	2159	2658	984	0	0	0
5	1092	8473	6271	6333	3786	225	0	0	0	0
6	1513	4932	5257	1233	2917	0	0	0	0	0
7	557	3463	6926	1368	0	0	0	0	0	0
8	1351	5596	6165	0	0	0	0	0	0	0
9	3133	2262	0	0	0	0	0	0	0	0
10	2063	0	0	0	0	0	0	0	0	0

1.10491735

	Reserve	Chain-L
	154.0	154
	617.4	617
	1,636.1	1636
	2,746.7	2747
1,363.0	3,169.1	3649
3,757.0	5,443.1	5435
9,814.6	12,220.0	10907
8,200.3	9,626.7	10650
22,344.6	24,905.4	16339
45,479.41	60,518.44	52135

Cumulative Percentages