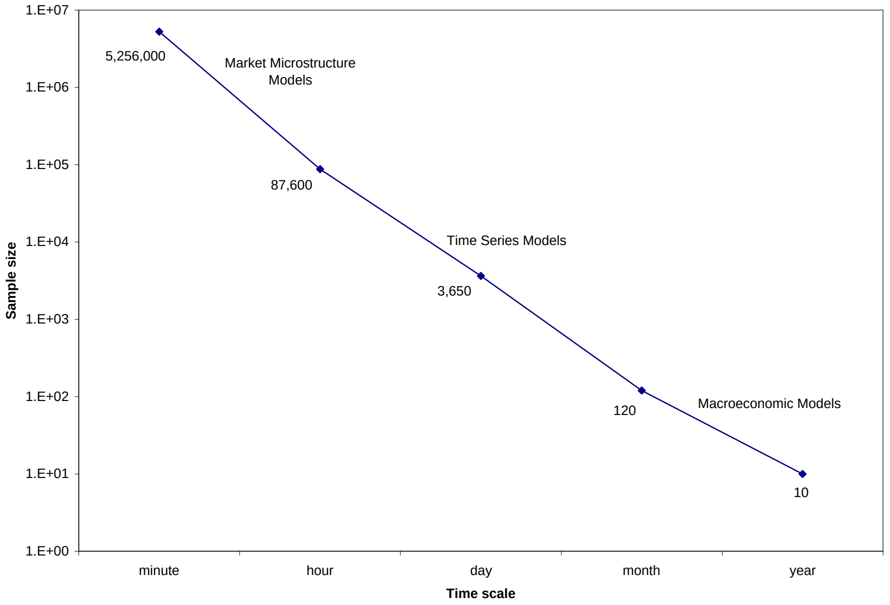
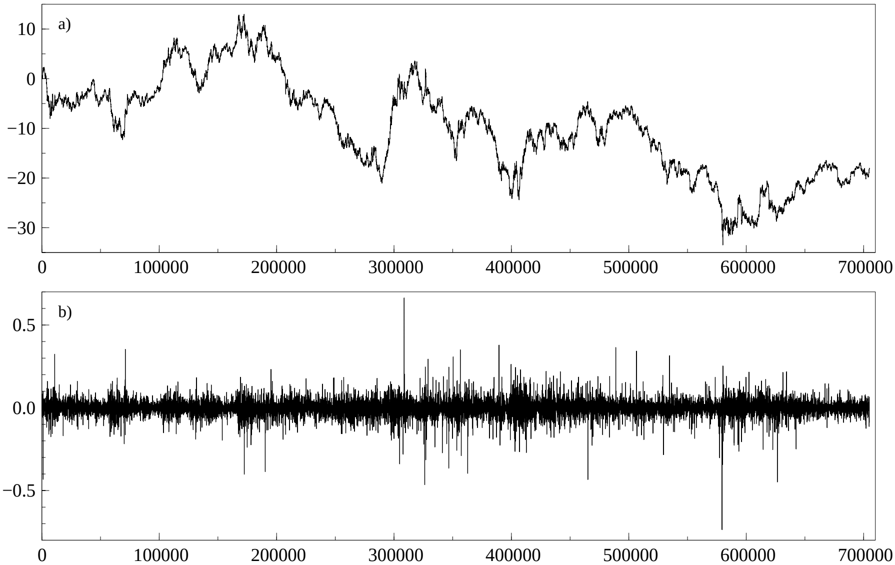
DETECTING JUMPS IN HIGH-FREQUENCY FINANCIAL SERIES USING MULTIPOWER VARIATION

Carla Ysusi

Instituto Tecnológico Autónomo de México

October 2006


1.E+07
1.E+06
1.E+05
1.E+04
1.E+03
1.E+02
1.E+01
1.E+00
Sample size
5,256,000
Market Microstructure Models
87,600
Time Series Models
3,650
120
Macroeconomic Models
10
minute
hour
day
month
year
Time scale



DOLLAR-DEUTSCHE MARK EXCHANGE RATE

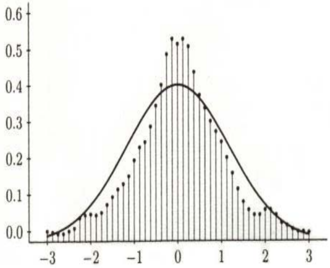
i.i.d. $y_t \sim N(0, \sigma^2)$ $\quad$ $y_t = \sigma \varepsilon_t \quad \varepsilon_t \sim N(0,1)$

PROBLEMS

1. Empirically y_{n+m}^2 and y_n^2 or $\mid y_{n+m} \mid$ and $\mid y_n \mid$ are correlated (Clustering phenomenom).

2. Not constant σ^2.

3. Unconditional distribution: Leptokurtosis (Heavy tails)



$y_t \sim$ Mixed Normal μ, σ^2 are random.

MODELS

AR,MA,ARMA	ARCH-GARCH	STOCHASTIC VOLATILITY
model μ	σ^2 not constant	Two sources of randomness
σ^2 constant	$y_t = \sigma_t \varepsilon_t$	ε_t, δ_t iid N(0,1) independent
	iid $\varepsilon_t \sim$N(0,1)	
	model cond. σ_t^2:	$y_t = \sigma_t \varepsilon_t$ $\sigma_t = f(\delta)$
	ARCH y_{t-1}^2 GARCH σ_{t-1}^2; y_{t-1}^2	Solving methods: simulation; numerical methods
	Engle(1982)	Clark(1973); Taylor(1982)

STOCHASTIC VOLATILITY MODEL

$$Y_t = \int_0^t a_u du + \int_0^t \sigma_s dW_s, \qquad t \geq 0$$

where $A_t = \int_0^t a_u du$.

Let σ_t and $A_t \perp W_t$. A_t is assumed to have locally bounded variation paths and it is set that $M_t = \int_0^t \sigma_s dW_s$, with the added condition that $\int_0^t \sigma_s^2 ds < \infty$ for all t. This is enough to guarantee that M_t is a local martingale.
So

$$Y_t = A_t + M_t.$$

Under these assumptions Y_t is a semimartingale. If additionally A_t is continuous then Y_t is a member of the continuous stochastic volatility semimartingale ($SVSM^c$) class.
Notice that σ_t can have serially dependent increments (clustering, fat tails) and long memory, and can allow jumps. Possible to include leverage effect.

QUADRATIC VARIATION

$$[Y]_t = p \lim_{n\to\infty} \sum_{j=0}^{n-1} (Y_{t_{j+1}} - Y_{t_j})^2,$$

where $t_0 = 0 < t_1 < \ldots < t_n = t$ with $sup_j\{t_{j+1} - t_j\} \to 0$ as $n \to \infty$.

As A_t is assumed to be continuous and of finite variation we obtain that

$$[Y]_t = [A]_t + 2[A, M]_t + [M]_t = [M]_t = \int_0^t \sigma_u^2 du = \sigma_t^{2*}$$

Define returns as

$$y_j = Y_{j\delta} - Y_{(j-1)\delta} \qquad j = 1, 2, 3, \ldots, \lfloor t/\delta \rfloor$$

QUADRATIC VARIATION

$$[Y]_t = p \lim_{n \to \infty} \sum_{j=0}^{n-1} (Y_{t_{j+1}} - Y_{t_j})^2,$$

where $t_0 = 0 < t_1 < \ldots < t_n = t$ with $sup_j\{t_{j+1} - t_j\} \to 0$ as $n \to \infty$.

As A_t is assumed to be continuous and of finite variation we obtain that

$$[Y]_t = [A]_t + 2[A, M]_t + [M]_t = [M]_t = \int_0^t \sigma_u^2 du = \sigma_t^{2*}$$

Define returns as

$$y_j = Y_{j\delta} - Y_{(j-1)\delta} \qquad j = 1, 2, 3, \ldots, \lfloor t/\delta \rfloor$$

REALISED VARIANCE

Definition

$$[Y_\delta]_t^{[2]} = \sum_{j=1}^{\lfloor t/\delta \rfloor} y_j^2$$

Andersen and Bollerslev (1998), Comte and Renault (1998) and Barndorff-Nielsen and Shephard (2001)

$$[Y_\delta]_t^{[2]} \xrightarrow{p} [Y]_t = \int_0^t \sigma_s^2 ds$$

if $Y \in SVSM^c$.

Barndorff-Nielsen, Graversen, Jacod, Podolskij and Shephard (2005) give result: when $\delta \downarrow 0$

$$\frac{\delta^{-1/2}([Y_\delta]_t^{[2]} - [Y]_t)}{\sqrt{2\int_0^t \sigma_s^4 ds}} \xrightarrow{L} N(0,1),$$

under the assumptions that A_t is of locally bounded variation, $\int_0^t \sigma_u^2 du < \infty$ and that σ_t is càdlàg.

REALISED VARIANCE

Definition

$$[Y_\delta]_t^{[2]} = \sum_{j=1}^{\lfloor t/\delta \rfloor} y_j^2$$

Andersen and Bollerslev (1998), Comte and Renault (1998) and Barndorff-Nielsen and Shephard (2001)

$$[Y_\delta]_t^{[2]} \xrightarrow{p} [Y]_t = \int_0^t \sigma_s^2 ds$$

if $Y \in SVSM^c$.
Barndorff-Nielsen, Graversen, Jacod, Podolskij and Shephard (2005) give result: when $\delta \downarrow 0$

$$\frac{\delta^{-1/2}([Y_\delta]_t^{[2]} - [Y]_t)}{\sqrt{2\int_0^t \sigma_s^4 ds}} \xrightarrow{L} N(0,1),$$

under the assumptions that A_t is of locally bounded variation, $\int_0^t \sigma_u^2 du < \infty$ and that σ_t is càdlàg.

INCLUDING JUMPS

$$Y_t = Y_t^{(1)} + Y_t^{(2)}$$

$$Y^{(1)} \in SVSM^c \qquad\qquad Y_t^{(2)} = \sum_{i=1}^{N_t} c_i$$

where c_i non-zero random variable and N_t finite activity simple counting process. Example: Compound Poisson Process.

Quadratic Variation

$$[Y]_t = \sigma_t^{2*} + \sum_{i=1}^{N_t} c_i^2 = [Y^{(1)}]_t + [Y^{(2)}]_t,$$

$$[Y_\delta]_t^{[2]} \xrightarrow{p} [Y]_t$$

INCLUDING JUMPS

$$Y_t = Y_t^{(1)} + Y_t^{(2)}$$

$$Y^{(1)} \in SVSM^c \qquad\qquad Y_t^{(2)} = \sum_{i=1}^{N_t} c_i$$

where c_i non-zero random variable and N_t finite activity simple counting process. Example: Compound Poisson Process.

Quadratic Variation

$$[Y]_t \;=\; \sigma_t^{2*} + \sum_{i=1}^{N_t} c_i^2 = [Y^{(1)}]_t + [Y^{(2)}]_t,$$

$$[Y_\delta]_t^{[2]} \xrightarrow{p} [Y]_t$$

Bipower Variation

$$\{Y_\delta\}_t^{[1,1]} = \sum_{j=1}^{\lfloor t/\delta \rfloor - 1} | y_j || y_{j+1} |$$

$$\mu_1^{-2}\{Y_\delta\}_t^{[1,1]} \xrightarrow{p} \int_0^t \sigma_s^2 ds \qquad \mu_r = E(| u |^r) \quad u \sim N(0,1).$$

TEST FOR JUMPS

$$[Y_\delta]_t^{[2]} - \mu_1^{-2}\{Y_\delta\}_t^{[1,1]} \xrightarrow{p} \sum_{i=1}^{N_t} c_i^2.$$

$$\frac{1}{\delta^{1/2}\sqrt{\int_0^t \sigma_s^4 ds}}\left([Y_\delta]_t^{[2]} - \mu_1^{-2}\{Y_\delta\}_t^{[1,1]}\right) \xrightarrow{L} N(0, \vartheta_{RV})$$

$\vartheta_{RV} \simeq 0.6091.$

Barndorff-Nielsen and Shephard(2004).

Andersen, Bollerslev and Diebold (2003), Barndorff-Nielsen and Shephard (2006), Huang and Tauchen (2005), Tauchen and Zhou (2006).

Bipower Variation

$$\{Y_\delta\}_t^{[1,1]} = \sum_{j=1}^{\lfloor t/\delta \rfloor - 1} | y_j || y_{j+1} |$$

$$\mu_1^{-2}\{Y_\delta\}_t^{[1,1]} \xrightarrow{p} \int_0^t \sigma_s^2 ds \qquad \mu_r = E(| u |^r) \quad u \sim N(0,1).$$

TEST FOR JUMPS

$$[Y_\delta]_t^{[2]} - \mu_1^{-2}\{Y_\delta\}_t^{[1,1]} \xrightarrow{p} \sum_{i=1}^{N_t} c_i^2.$$

$$\frac{1}{\delta^{1/2}\sqrt{\int_0^t \sigma_s^4 ds}}\left([Y_\delta]_t^{[2]} - \mu_1^{-2}\{Y_\delta\}_t^{[1,1]}\right) \xrightarrow{L} N(0, \vartheta_{RV})$$

$\vartheta_{RV} \simeq 0.6091.$

Barndorff-Nielsen and Shephard(2004).

Andersen, Bollerslev and Diebold (2003), Barndorff-Nielsen and Shephard (2006), Huang and Tauchen (2005), Tauchen and Zhou (2006).

TRIPOWER VARIATION

$$\{Y_\delta\}_t^{[2/3,2/3,2/3]} = \sum_{j=1}^{\lfloor t/\delta \rfloor - 2} \mid y_j \mid^{2/3} \mid y_{j+1} \mid^{2/3} \mid y_{j+2} \mid^{2/3}$$

$$\mu_{2/3}^{-3} \{Y_\delta\}_t^{[2/3,2/3,2/3]} \xrightarrow{p} \int_0^t \sigma_s^2 ds,$$

$$[Y_\delta]_t^{[2]} - \mu_{2/3}^{-3} \{Y_\delta\}_t^{[2/3,2/3,2/3]} \xrightarrow{p} \sum_{i=1}^{N_t} c_i^2$$

QUADPOWER VARIATION

$$\{Y_\delta\}_t^{[1/2,1/2,1/2,1/2]} = \sum_{j=1}^{\lfloor t/\delta \rfloor - 3} \mid y_j \mid^{1/2} \mid y_{j+1} \mid^{1/2} \mid y_{j+2} \mid^{1/2} \mid y_{j+3} \mid^{1/2}$$

$$\mu_{1/2}^{-4} \{Y_\delta\}_t^{[1/2,1/2,1/2,1/2]} \xrightarrow{p} \int_0^t \sigma_s^2 ds.$$

$$[Y_\delta]_t^{[2]} - \mu_{1/2}^{-4} \{Y_\delta\}_t^{[1/2,1/2,1/2,1/2]} \xrightarrow{p} \sum_{i=1}^{N_t} c_i^2.$$

BIPOWER VARIATION

$$\frac{1}{\delta^{1/2}\sqrt{\int_0^t \sigma_u^4 du}} \begin{pmatrix} \sum_{j=1}^{t/\delta-2} y_j^2 - \int_0^t \sigma_u^2 du \\ \mu_1^{-2} \sum_{j=1}^{t/\delta-2} \mid y_j \mid\mid y_{j+1} \mid - \int_0^t \sigma_u^2 du \end{pmatrix} \to N\left(0, \quad \begin{pmatrix} 2 & 2 \\ 2 & 2.60907 \end{pmatrix}\right)$$

SKIPPED VERSION BIPOWER VARIATION

$$\frac{1}{\delta^{1/2}\sqrt{\int_0^t \sigma_u^4 du}} \begin{pmatrix} \sum_{j=1}^{t/\delta-2} y_j^2 - \int_0^t \sigma_u^2 du \\ \mu_1^{-2} \sum_{j=1}^{t/\delta-2} \mid y_j \mid\mid y_{j+2} \mid - \int_0^t \sigma_u^2 du \end{pmatrix} \to N\left(0, \quad \begin{pmatrix} 2 & 2 \\ 2 & 2.60907 \end{pmatrix}\right)$$

TRIPOWER VARIATION

$$\frac{1}{\delta^{1/2}\sqrt{\int_0^t \sigma^4(u) du}} \begin{pmatrix} \sum_{j=1}^{t/\delta-2} y_j^2 - \int_0^t \sigma_u^2 du \\ \mu_{2/3}^{-3} \sum_{j=1}^{t/\delta-2} \mid y_j \mid^{2/3}\mid y_{j+1} \mid^{2/3}\mid y_{j+2} \mid^{2/3} - \int_0^t \sigma_u^2 du \end{pmatrix}$$

$$\to N\left(0, \quad \begin{pmatrix} 2 & 2 \\ 2 & 3.0613 \end{pmatrix}\right)$$

QUADPOWER VARIATION

$$\frac{1}{\delta^{1/2}\sqrt{\int_0^t \sigma_u^4 du}} \begin{pmatrix} \sum_{j=1}^{t/\delta-3} y_j^2 - \int_0^t \sigma_u^2 du \\ \mu_{1/2}^{-4} \sum_{j=1}^{t/\delta-3} \sqrt{\mid y_{j,i} \mid\mid y_{j+1,i} \mid\mid y_{j+2,i} \mid\mid y_{j+3,i} \mid} - \int_0^t \sigma_u^2 du \end{pmatrix}$$

$$\to N\left(0, \quad \begin{pmatrix} 2 & 2 \\ 2 & 3.3704 \end{pmatrix}\right)$$

TESTS FOR JUMPS

H_0: The price process does not include a jump component.
H_1: The prices process consists of a continuous and jump component.

Assume the jump component is a finite activity jump process.

Linear Test-Statistics

$$\frac{1}{\delta^{1/2}\sqrt{\int_0^t \sigma_s^4 ds}}\left(\mu_{2/3}^{-3}\{Y_\delta\}_t^{[2/3,2/3,2/3]} - [Y_\delta]_t^{[2]}\right) \xrightarrow{L} N(0, \vartheta_{TV})$$

where $\vartheta_{TV} = \mu_{4/3}\mu_{2/3}^{-2}(\mu_{4/3}^2\mu_{2/3}^{-4} + 2\mu_{4/3}\mu_{2/3}^{-2} - 2) - 7 \simeq 1.0613$

$$\frac{1}{\delta^{1/2}\sqrt{\int_0^t \sigma_s^4 ds}}\left(\mu_{1/2}^{-4}\{Y_\delta\}_t^{[1/2,1/2,1/2,1/2]} - [Y_\delta]_t^{[2]}\right) \xrightarrow{L} N(0, \vartheta_{QV})$$

where $\vartheta_{QV} = \mu_1\mu_{1/2}^{-2}(\mu_1^3\mu_{1/2}^{-6} + 2\mu_1^2\mu_{1/2}^{-4} + 2\mu_1\mu_{1/2}^{-2} - 2) - 9 \simeq 1.37702$

$$\frac{1}{\delta^{1/2}\sqrt{\int_0^t \sigma_s^4 ds}}\left(\mu_1^{-2}\{Y_\delta\}_t^{[1,0,1]} - [Y_\delta]_t^{[2]}\right) \xrightarrow{L} N(0, \vartheta_{SBV})$$

where$\vartheta_{SBV} = \mu_1^{-4} + 2\mu_1^{-2} - 5 \simeq 0.60907$

Ratio Test-Statistics

$$\frac{\left(\frac{\mu_{2/3}^{-3}\{Y_\delta\}_t^{[2/3,2/3,2/3]}}{[Y_\delta]_t^{[2]}} - 1\right)}{\delta^{1/2}\sqrt{\frac{\int_0^t \sigma_s^4 ds}{\left(\int_0^t \sigma_s^2 ds\right)^2}}} \xrightarrow{L} N(0, \vartheta_{TV})$$

where $\vartheta_{TV} \simeq 1.0613$;

$$\frac{\left(\frac{\mu_{1/2}^{-4}\{Y_\delta\}_t^{[1/2,1/2,1/2,1/2]}}{[Y_\delta]_t^{[2]}} - 1\right)}{\delta^{1/2}\sqrt{\frac{\int_0^t \sigma_s^4 ds}{\left(\int_0^t \sigma_s^2 ds\right)^2}}} \xrightarrow{L} N(0, \vartheta_{QV})$$

where $\vartheta_{QV} \simeq 1.37702$;

$$\frac{\left(\frac{\mu_1^{-2}\{Y_\delta\}_t^{[1,0,1]}}{[Y_\delta]_t^{[2]}} - 1\right)}{\delta^{1/2}\sqrt{\frac{\int_0^t \sigma_s^4 ds}{\left(\int_0^t \sigma_s^2 ds\right)^2}}} \xrightarrow{L} N(0, \vartheta_{SBV})$$

where $\vartheta_{SBV} \simeq 0.60907$.

Integrated Quarticity

1) Realised Quarticity (E1)

$$M\mu_4^{-1}\{Y_M\}_i^{[4]} = M\mu_4^{-1}\sum_{j=1}^{M} y_{j,i}^4$$

2) Realised Tripower Variation with $r = s = u = 4/3$ (E2)

$$M\mu_{4/3}^{-3}\{Y_M\}_i^{[4/3,4/3,4/3]} = M\mu_{4/3}^{-3}\sum_{j=1}^{M-2} \mid y_{j,i} \mid^{4/3}\mid y_{j+1,i} \mid^{4/3}\mid y_{j+2,i} \mid^{4/3}$$

3) Realised Quadpower Variation with $r = s = u = v = 1$ (E3)

$$M\mu_1^{-4}\{Y_M\}_i^{[1,1,1,1]} = M\mu_1^{-4}\sum_{j=1}^{M-3} \mid y_{j,i} \mid\mid y_{j+1,i} \mid\mid y_{j+2,i} \mid\mid y_{j+3,i} \mid.$$

One-sided test

$$min\left(0, \quad \mu_{2/3}^{-3}\{Y_M\}_i^{[2/3,2/3,2/3]} - [Y_M]_i^{[2]}\right)$$

Modified estimators

$$\left(\frac{M}{M-2}\right)\mu_{2/3}^{-3}\{Y_M\}_i^{[2/3,2/3,2/3]}, \qquad \left(\frac{M}{M-3}\right)\mu_{1/2}^{-4}\{Y_M\}_i^{[1/2,1/2,1/2,1/2]},$$

$$\left(\frac{M}{M-2}\right)\mu_1^{-2}\{Y_M\}_i^{[1,0,1]}.$$

Integrated Quarticity

1) Realised Quarticity (E1)

$$M\mu_4^{-1}\{Y_M\}_i^{[4]} = M\mu_4^{-1}\sum_{j=1}^{M} y_{j,i}^4$$

2) Realised Tripower Variation with $r = s = u = 4/3$ (E2)

$$M\mu_{4/3}^{-3}\{Y_M\}_i^{[4/3,4/3,4/3]} = M\mu_{4/3}^{-3}\sum_{j=1}^{M-2} \mid y_{j,i} \mid^{4/3} \mid y_{j+1,i} \mid^{4/3} \mid y_{j+2,i} \mid^{4/3}$$

3) Realised Quadpower Variation with $r = s = u = v = 1$ (E3)

$$M\mu_1^{-4}\{Y_M\}_i^{[1,1,1,1]} = M\mu_1^{-4}\sum_{j=1}^{M-3} \mid y_{j,i} \mid\mid y_{j+1,i} \mid\mid y_{j+2,i} \mid\mid y_{j+3,i} \mid .$$

One-sided test

$$min\left(0, \quad \mu_{2/3}^{-3}\{Y_M\}_i^{[2/3,2/3,2/3]} - [Y_M]_i^{[2]}\right)$$

Modified estimators

$$\left(\frac{M}{M-2}\right)\mu_{2/3}^{-3}\{Y_M\}_i^{[2/3,2/3,2/3]}, \qquad \left(\frac{M}{M-3}\right)\mu_{1/2}^{-4}\{Y_M\}_i^{[1/2,1/2,1/2,1/2]},$$

$$\left(\frac{M}{M-2}\right)\mu_1^{-2}\{Y_M\}_i^{[1,0,1]}.$$

Integrated Quarticity

1) Realised Quarticity (E1)

$$M\mu_4^{-1}\{Y_M\}_i^{[4]} = M\mu_4^{-1}\sum_{j=1}^{M} y_{j,i}^4$$

2) Realised Tripower Variation with $r = s = u = 4/3$ (E2)

$$M\mu_{4/3}^{-3}\{Y_M\}_i^{[4/3,4/3,4/3]} = M\mu_{4/3}^{-3}\sum_{j=1}^{M-2} \mid y_{j,i} \mid^{4/3}\mid y_{j+1,i} \mid^{4/3}\mid y_{j+2,i} \mid^{4/3}$$

3) Realised Quadpower Variation with $r = s = u = v = 1$ (E3)

$$M\mu_1^{-4}\{Y_M\}_i^{[1,1,1,1]} = M\mu_1^{-4}\sum_{j=1}^{M-3} \mid y_{j,i} \mid\mid y_{j+1,i} \mid\mid y_{j+2,i} \mid\mid y_{j+3,i} \mid.$$

One-sided test

$$min\left(0, \quad \mu_{2/3}^{-3}\{Y_M\}_i^{[2/3,2/3,2/3]} - [Y_M]_i^{[2]}\right)$$

Modified estimators

$$\left(\frac{M}{M-2}\right)\mu_{2/3}^{-3}\{Y_M\}_i^{[2/3,2/3,2/3]}, \qquad \left(\frac{M}{M-3}\right)\mu_{1/2}^{-4}\{Y_M\}_i^{[1/2,1/2,1/2,1/2]},$$

$$\left(\frac{M}{M-2}\right)\mu_1^{-2}\{Y_M\}_i^{[1,0,1]}.$$

Simulations

Square Root Process (Cox, Ingersoll and Ross (1985))

$$d\sigma_t^2 = -\lambda\{\sigma_t^2 - \xi\}dt + \omega\sigma_t dB_{\lambda t}, \qquad \xi \geq \omega^2/2, \qquad \lambda > 0,$$

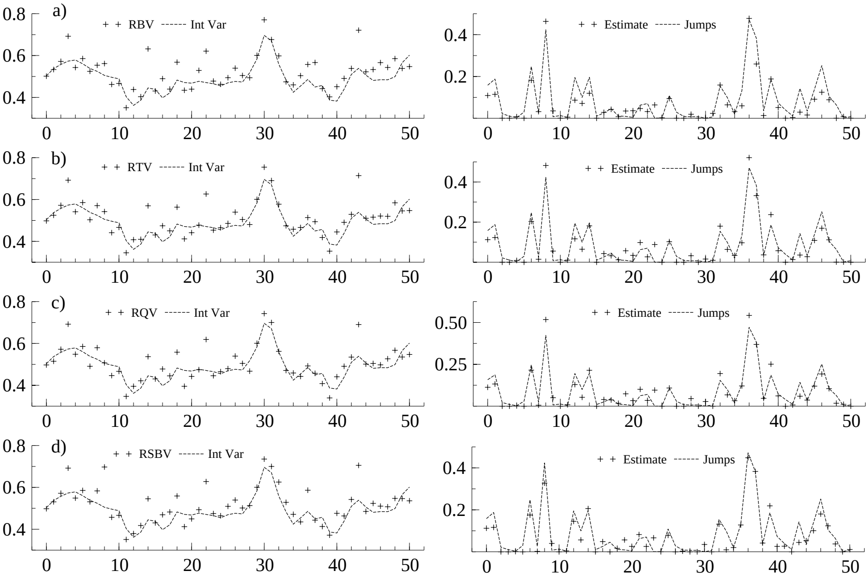
where B is a standard Brownian motion process.

The square root process has a marginal distribution

$$\sigma_t^2 \sim \Gamma(2\omega^{-2}\xi, 2\omega^{-2}) = \Gamma(\nu, a), \qquad \nu \geq 1,$$

with a mean of $\xi = \nu/a$ and a variance of $\omega^2 = \nu/a^2$.

We will take $A_t = 0$ and rule out the leverage effect by assuming $Cor\{B_{\lambda t}, W_t\} = 0$. We will take $h = 1$, $\lambda = 0.01$, $\nu = 4$ and $a = 8$. The jumps will be i.i.d. $N(0, \beta\nu/a)$, thus a jump has the same variance as that expected over a $(\beta \times 100)\%$ of a day when there are no jumps.


a)
RBV
Int Var
Estimate
Jumps
b)
RTV
Int Var
Estimate
Jumps
c)
RQV
Int Var
Estimate
Jumps
d)
RSBV
Int Var
Estimate
Jumps

NO JUMPS ADDED

Infeasible Tests

M	12			72			288		
	Bias	SD	Cove	Bias	SD	Cove	Bias	SD	Cove
RBV	−.363	1.16	87.4	−.191	1.00	92.8	−.115	1.00	93.5
RTV	−.313	1.13	88.5	−.180	1.00	92.9	−.103	.994	94.1
RQV	−.313	1.14	89.0	−.181	1.00	93.3	−.104	.999	94.3
RSBV	−.424	1.22	85.9	−.199	1.03	92.2	−.129	1.01	93.6

Bias, standard deviation and coverage (95% level) of the infeasible linear test.

Feasible Tests

	E1			E2			E3		
	Bias	SD	Cove	Bias	SD	Cove	Bias	SD	Cove
RBV	−.086	.997	93.5	−.165	1.03	91.6	−.173	1.04	91.3
RTV	−.075	.992	94.3	−.154	1.02	92.1	−.167	1.03	91.6
RQV	−.076	.999	94.8	−.149	1.02	92.8	−.167	1.04	91.7
RSBV	−.103	1.01	94.4	−.161	1.04	92.9	−.175	1.05	92.2

Bias, standard deviation and coverage (95% level) of the feasible linear test for M=288.

	E1			E2			E3		
	Bias	SD	Cove	Bias	SD	Cove	Bias	SD	Cove
RBV	−.041	.986	94.9	−.110	.980	93.7	−.113	.980	93.6
RTV	−.015	.987	96.4	−.094	.971	94.1	−.099	.976	94.1
RQV	−.007	.998	97.1	−.087	.968	94.8	−.097	.980	94.4
RSBV	−.056	.994	95.0	−.117	.985	94.1	−.124	.989	94.0

Bias, standard deviation and coverage (95% level) of the feasible ratio test with M=288.

ADDED JUMP COMPONENT

Infeasible

M			12			72			288	
		Bias	SD	Cove	Bias	SD	Cove	Bias	SD	Cove
50%	RBV	−.728	1.75	78.1	−1.55	3.21	69.8	−3.49	6.42	55.0
	RTV	−.639	1.60	79.5	−1.36	2.70	70.5	−2.91	5.19	56.5
	RQV	−.623	1.55	79.7	−1.27	2.47	71.1	−2.65	4.66	57.7
	RSBV	−.796	1.82	76.9	−1.54	3.05	69.2	−3.51	6.48	54.9
20%	RBV	−.469	1.25	85.2	−.190	1.17	89.9	−.389	1.49	87.3
	RTV	−.407	1.23	86.2	−.143	1.13	91.4	−.327	1.35	88.5
	RQV	−.384	1.22	86.8	−.139	1.14	90.7	−.291	1.28	90.1
	RSBV	−.505	1.28	83.9	−.224	1.15	90.6	−.399	1.46	87.0

Bias, standard deviation and coverage (95% level) of the infeasible linear test.

Feasible

			E1			E2			E3	
		Bias	SD	Cove	Bias	SD	Cove	Bias	SD	Cove
50%	RBV	−.667	.875	92.8	−2.10	2.88	58.2	−2.16	2.96	57.7
	RTV	−.549	.808	96.6	−1.72	2.32	60.6	−1.79	2.42	59.9
	RQV	−.498	.782	97.6	−1.54	2.07	62.4	−1.61	2.17	61.1
	RSBV	−.656	.877	93.3	−2.07	2.87	59.2	−2.14	2.97	58.5
20%	RBV	−.185	.956	94.2	−.328	1.22	88.4	−.331	1.23	88.3
	RTV	−.143	.918	96.6	−.267	1.13	90.6	−.274	1.13	90.4
	RQV	−.117	.906	97.3	−.228	1.08	91.9	−.239	1.02	91.3
	RSBV	−.203	.924	95.8	−.327	1.16	88.7	−.337	1.18	88.5

Bias, standard deviation and coverage (95% level) of the feasible ratio test when one jump is added every day for M=288.

SIZE ADJUSTED TESTS

Infeasible

M		12				72				288		
	CV	NQ	Pow	Pow	CV	NQ	Pow	Pow	CV	NQ	Pow	Pow
β			50%	20%			50%	20%			50%	20%
BV	2.46	99.3	0.09	0.06	1.93	97.3	0.20	0.06	1.85	96.8	0.34	0.09
TV	2.27	98.8	0.10	0.06	1.84	96.7	0.20	0.06	1.75	96.1	0.35	0.08
QV	2.09	98.2	0.11	0.07	1.80	96.4	0.18	0.07	1.71	95.7	0.34	0.08
SBV	2.63	99.6	0.09	0.06	1.88	97.1	0.20	0.06	1.83	96.7	0.35	0.08

Critical values (5% size), Normal quantiles and power of the infeasible linear tests when one jump is added every day.

Feasible

		E1				E2				E3		
	CV	NQ	Pow	Pow	CV	NQ	Pow	Pow	CV	NQ	Pow	Pow
β			50%	20%			50%	20%			50%	20%
BV	1.63	94.9	0.08	0.05	1.79	96.4	0.33	0.08	1.80	96.5	0.33	0.08
TV	1.52	93.6	0.07	0.04	1.73	95.9	0.32	0.06	1.75	96.1	0.32	0.06
QV	1.46	92.8	0.06	0.05	1.65	95.2	0.31	0.06	1.70	95.6	0.31	0.06
SBV	1.63	94.9	0.09	0.05	1.78	96.3	0.33	0.07	1.77	96.2	0.34	0.07

Critical values, Normal quantiles and power of the feasible ratio tests for M=288 when one jump is added every day.

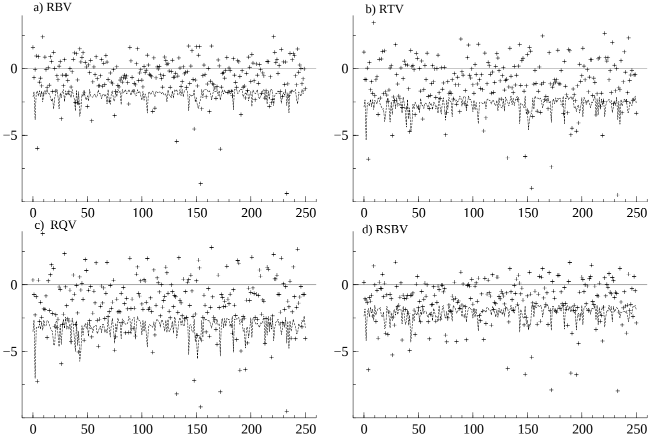
Empirical Data

Dollar/Deutsche Mark exchange rates. (Olsen and Associates)

From 1st of December 1986 until 30th of November 1996

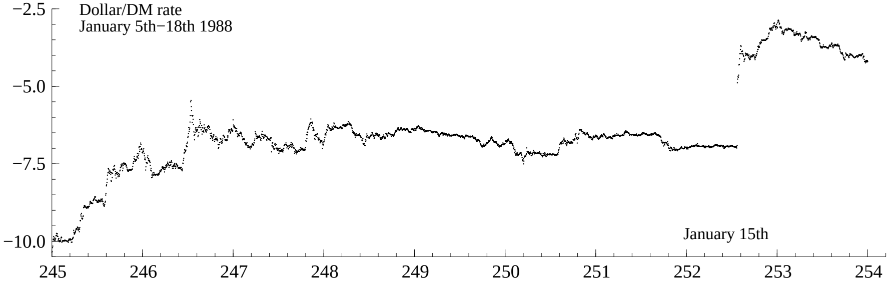
Quotes from Reuters screen

Five-minutes data (Market Microstructure Effect)

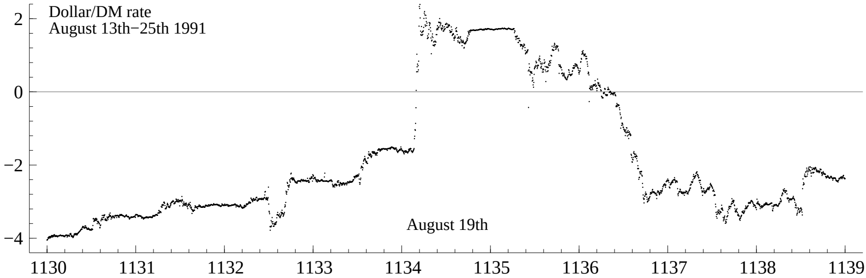


M	corr	RV	BV	5%	1%	TV	5%	1%	QV	5%	1%	SB	5%	1%
12	.001	.46	.39	.18	.08	.36	.16	.04	.34	.14	.02	.36	.22	.10
24	.002	.46	.40	.21	.10	.37	.19	.08	.35	.18	.05	.37	.23	.12
72	−.001	.49	.44	.22	.12	.42	.23	.11	.40	.22	.09	.42	.28	.16
144	−.056	.51	.47	.23	.12	.45	.25	.13	.43	.26	.11	.44	.34	.19
288	−.092	.53	.50	.19	.10	.48	.23	.11	.47	.25	.11	.47	.37	.21

Sum of the first five correlation coefficients of the Dollar/ DM series (corr). Average value of realised variance (RV), realised bipower (BV), tripower (TV), quadpower (QV) and skipped bipower (SB) variation. Proportion of rejections of the null hypothesis at the 5% and 1% level.



−2.5
Dollar/DM rate
January 5th−18th 1988
−5.0
−7.5
January 15th
−10.0
245
246
247
248
249
250
251
252
253
254



Dollar/DM rate
August 13th−25th 1991
2
0
−2
August 19th
−4
1130
1131
1132
1133
1134
1135
1136
1137
1138
1139

CONCLUSIONS

Multipower variation provides a tool to test for the presence of jumps in log-price process.

Tests based on Tripower and Quadpower variation seem to have better size.

Tripower or Quadpower variation give better estimations of integrated quarticity.

Study the effect of Market Microstructure Noise.