DETECTING JUMPS IN HIGH-FREQUENCY FINANCIAL SERIES USING MULTIPOWER VARIATION

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Market Microstructure Models

Time Series Models

Macroeconomic Models

Sample size vs. Time scale

- Minute: 87,600
- Hour: 3,650
- Day: 120
- Month: 10
- Year: 1,000
- Month: 1
- Year: 1
DOLLAR-DEUTSCHE MARK EXCHANGE RATE
i.i.d. \( y_t \sim N(0, \sigma^2) \) \[ y_t = \sigma \varepsilon_t \quad \varepsilon_t \sim N(0, 1) \]

**PROBLEMS**

1. Empirically \( y_{n+m}^2 \) and \( y_n^2 \) or \( |y_{n+m}| \) and \( |y_n| \) are correlated (Clustering phenomenon).

2. Not constant \( \sigma^2 \).

3. Unconditional distribution: Leptokurtosis (Heavy tails)
\( y_t \sim \text{Mixed Normal } \mu, \sigma^2 \text{ are random.} \)

### MODELS

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<td>model ( \mu )</td>
<td>( \sigma^2 \text{ not constant} )</td>
<td>Two sources of randomness</td>
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<td>( \sigma^2 \text{ constant} )</td>
<td>( y_t = \sigma_t \varepsilon_t )</td>
<td>( \varepsilon_t, \delta_t \text{ iid } \mathcal{N}(0,1) ) independent</td>
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<td>model cond. ( \sigma^2_t ):</td>
<td>iid ( \varepsilon_t \sim \mathcal{N}(0,1) )</td>
<td>( y_t = \sigma_t \varepsilon_t )</td>
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<td>ARCH ( y_{t-1}^2 )</td>
<td>( \sigma_t = f(\delta) )</td>
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<td>GARCH ( \sigma^2_{t-1}; y_{t-1}^2 )</td>
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STOCHASTIC VOLATILITY MODEL

\[ Y_t = \int_0^t a_u du + \int_0^t \sigma_s dW_s, \quad t \geq 0 \]

where \( A_t = \int_0^t a_u du \).

Let \( \sigma_t \) and \( A_t \perp W_t \). \( A_t \) is assumed to have locally bounded variation paths and it is set that \( M_t = \int_0^t \sigma_s dW_s \), with the added condition that \( \int_0^t \sigma_s^2 ds < \infty \) for all \( t \). This is enough to guarantee that \( M_t \) is a local martingale.

So

\[ Y_t = A_t + M_t. \]

Under these assumptions \( Y_t \) is a semimartingale. If additionally \( A_t \) is continuous then \( Y_t \) is a member of the continuous stochastic volatility semimartingale (SVSM⁰) class.

Notice that \( \sigma_t \) can have serially dependent increments (clustering, fat tails) and long memory, and can allow jumps. Possible to include leverage effect.
QUADRATIC VARIATION

\[ [Y]_t = p \lim_{n \to \infty} \sum_{j=0}^{n-1} (Y_{t_{j+1}} - Y_{t_j})^2, \]

where \( t_0 = 0 < t_1 < \ldots < t_n = t \) with \( \sup_j \{t_{j+1} - t_j\} \to 0 \) as \( n \to \infty \).

As \( A_t \) is assumed to be continuous and of finite variation we obtain that

\[ [Y]_t = [A]_t + 2[A, M]_t + [M]_t = [M]_t = \int_0^t \sigma_u^2 du = \sigma_t^{2*} \]

Define returns as

\[ y_j = Y_{j\delta} - Y_{(j-1)\delta} \quad \text{for} \quad j = 1, 2, 3, \ldots, \lfloor t/\delta \rfloor \]
QUADRATIC VARIATION

\[ [Y]_t = \lim_{n \to \infty} \sum_{j=0}^{n-1} (Y_{t_{j+1}} - Y_{t_j})^2, \]

where \( t_0 = 0 < t_1 < \ldots < t_n = t \) with \( \sup_j \{t_{j+1} - t_j\} \to 0 \) as \( n \to \infty \).

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\[ [Y]_t = [A]_t + 2[A, M]_t + [M]_t = [M]_t = \int_0^t \sigma_u^2 du = \sigma_t^{2*} \]

Define returns as

\[ y_j = Y_{j\delta} - Y_{(j-1)\delta} \quad j = 1, 2, 3, \ldots, \lfloor t/\delta \rfloor \]
REALISED VARIANCE

Definition

\[
[Y_\delta]_t^{[2]} = \left\lfloor \frac{t}{\delta} \right\rfloor \sum_{j=1}^{t/\delta} y_j^2
\]


\[
[Y_\delta]_t^{[2]} \overset{p}{\rightarrow} [Y]_t = \int_0^t \sigma_s^2 ds
\]

if \( Y \in SVSM_c \).

Barndorff-Nielsen, Graversen, Jacod, Podolskij and Shephard (2005) give result: when \( \delta \downarrow 0 \)

\[
\frac{\delta^{-1/2}([Y_\delta]_t^{[2]} - [Y]_t)}{\sqrt{2 \int_0^t \sigma_s^4 ds}} \overset{L}{\rightarrow} N(0, 1),
\]

under the assumptions that \( A_t \) is of locally bounded variation, \( \int_0^t \sigma_u^2 du < \infty \) and that \( \sigma_t \) is càdlàg.
REALISED VARIANCE

Definition

\[
[Y_\delta]_t^{[2]} = \sum_{j=1}^{\lfloor t/\delta \rfloor} y_j^2
\]


\[
[Y_\delta]_t^{[2]} \xrightarrow{p} [Y]_t = \int_0^t \sigma_s^2 ds
\]

if \( Y \in SVSM^c \).

Barndorff-Nielsen, Graversen, Jacod, Podolskij and Shephard (2005) give result: when \( \delta \downarrow 0 \)

\[
\frac{\delta^{-1/2}([Y_\delta]_t^{[2]} - [Y]_t)}{\sqrt{2 \int_0^t \sigma_s^4 ds}} \xrightarrow{L} \mathcal{N}(0, 1),
\]

under the assumptions that \( A_t \) is of locally bounded variation, \( \int_0^t \sigma_u^2 du < \infty \) and that \( \sigma_t \) is càdlàg.
INCLUDING JUMPS

\[ Y_t = Y_t^{(1)} + Y_t^{(2)} \]

\[ Y^{(1)} \in SVSM^c \quad Y_t^{(2)} = \sum_{i=1}^{N_t} c_i \]

where \( c_i \) non-zero random variable and \( N_t \) finite activity simple counting process. Example: Compound Poisson Process.

Quadratic Variation

\[ [Y]_t = \sigma_t^{2*} + \sum_{i=1}^{N_t} c_i^2 = [Y^{(1)}]_t + [Y^{(2)}]_t, \]

\[ [Y_\delta]_t^{[2]} \xrightarrow{P} [Y]_t \]
INCLUDING JUMPS

\[ Y_t = Y_t^{(1)} + Y_t^{(2)} \]

\[ Y^{(1)} \in SVSM^c \quad \quad \quad Y_t^{(2)} = \sum_{i=1}^{N_t} c_i \]

where \( c_i \) non-zero random variable and \( N_t \) finite activity simple counting process. Example: Compound Poisson Process.

**Quadratic Variation**

\[ [Y]_t = \sigma_t^{2*} + \sum_{i=1}^{N_t} c_i^2 = [Y^{(1)}]_t + [Y^{(2)}]_t, \]

\[ [Y_\delta]_t^{[2]} \xrightarrow{P} [Y]_t \]
Bipower Variation

\[
\{Y_\delta\}_t^{[1,1]} = \left\lfloor \frac{t}{\delta} \right\rfloor^{-1} \sum_{j=1}^{\left\lfloor \frac{t}{\delta} \right\rfloor - 1} |Y_j| |Y_{j+1}|
\]

\[
\mu_1^{-2} \{Y_\delta\}_t^{[1,1]} \overset{P}{\rightarrow} \int_0^t \sigma_s^2 ds \quad \mu_r = E(|u|^r) \quad u \sim N(0, 1).
\]

TEST FOR JUMPS

\[
[Y_\delta]_t^{[2]} - \mu_1^{-2} \{Y_\delta\}_t^{[1,1]} \overset{P}{\rightarrow} \sum_{i=1}^{N_t} c_i^2.
\]

\[
\frac{1}{\delta^{1/2} \sqrt{\int_0^t \sigma_s^4 ds}} (\left[ Y_\delta \right]_t^{[2]} - \mu_1^{-2} \{Y_\delta\}_t^{[1,1]}) \overset{L}{\rightarrow} N(0, \vartheta_{RV})
\]

\[
\vartheta_{RV} \approx 0.6091.
\]


Bipower Variation

\[
\{ Y_\delta \}^{[1,1]}_t = \sum_{j=1}^{\lfloor t/\delta \rfloor} \left| Y_j \right| \left| Y_{j+1} \right|
\]

\[
\mu_1^{-2} \{ Y_\delta \}^{[1,1]}_t \xrightarrow{p} \int_0^t \sigma_s^2 ds \quad \mu_r = E(|u|^r) \quad u \sim N(0, 1).
\]

TEST FOR JUMPS

\[
[Y_\delta]^{[2]}_t - \mu_1^{-2} \{ Y_\delta \}^{[1,1]}_t \xrightarrow{p} \sum_{i=1}^{N_t} c_i^2.
\]

\[
\frac{1}{\delta^{1/2} \sqrt{\int_0^t \sigma_s^4 ds}} \left( [Y_\delta]^{[2]}_t - \mu_1^{-2} \{ Y_\delta \}^{[1,1]}_t \right) \xrightarrow{L} N(0, \vartheta_{RV})
\]

\[
\vartheta_{RV} \approx 0.6091.
\]


TRIPOWER VARIATION

\[
\{ Y_\delta \}_{t}^{[2/3,2/3,2/3]} = \sum_{j=1}^{\lfloor t/\delta \rfloor - 2} | y_j |^{2/3} | y_{j+1} |^{2/3} | y_{j+2} |^{2/3}
\]

\[
\mu_{2/3}^{-3} \{ Y_\delta \}_{t}^{[2/3,2/3,2/3]} \xrightarrow{p} \int_{0}^{t} \sigma_s^2 ds,
\]

\[
[Y_\delta]_{t}^{[2]} - \mu_{2/3}^{-3} \{ Y_\delta \}_{t}^{[2/3,2/3,2/3]} \xrightarrow{p} \sum_{i=1}^{N_t} c_i^2
\]

QUADPOWER VARIATION

\[
\{ Y_\delta \}_{t}^{[1/2,1/2,1/2,1/2]} = \sum_{j=1}^{\lfloor t/\delta \rfloor - 3} | y_j |^{1/2} | y_{j+1} |^{1/2} | y_{j+2} |^{1/2} | y_{j+3} |^{1/2}
\]

\[
\mu_{1/2}^{-4} \{ Y_\delta \}_{t}^{[1/2,1/2,1/2,1/2]} \xrightarrow{p} \int_{0}^{t} \sigma_s^2 ds.
\]

\[
[Y_\delta]_{t}^{[2]} - \mu_{1/2}^{-4} \{ Y_\delta \}_{t}^{[1/2,1/2,1/2,1/2]} \xrightarrow{p} \sum_{i=1}^{N_t} c_i^2.
\]
BIPOWER VARIATION

\[
\frac{1}{\delta^{1/2} \sqrt{\int_0^t \sigma_u^4 du}} \left( \mu_1^{-2} \sum_{j=1}^{t/\delta-2} y_j^2 - \int_0^t \sigma_u^2 du \right) \rightarrow N \left( 0, \left( \begin{array}{cc} 2 & 2 \\ 2 & 2.60907 \end{array} \right) \right)
\]

SKIPPED VERSION BIPOWER VARIATION

\[
\frac{1}{\delta^{1/2} \sqrt{\int_0^t \sigma_u^4 du}} \left( \mu_1^{-2} \sum_{j=1}^{t/\delta-2} |y_j||y_{j+1}| - \int_0^t \sigma_u^2 du \right) \rightarrow N \left( 0, \left( \begin{array}{cc} 2 & 2 \\ 2 & 2.60907 \end{array} \right) \right)
\]

TRIPOWER VARIATION

\[
\frac{1}{\delta^{1/2} \sqrt{\int_0^t \sigma_u^4(u) du}} \left( \mu_2^{-3/2} \sum_{j=1}^{t/\delta-2} |y_j|^{2/3} |y_{j+1}|^{2/3} |y_{j+2}|^{2/3} - \int_0^t \sigma_u^2 du \right) \rightarrow N \left( 0, \left( \begin{array}{cc} 2 & 2 \\ 2 & 3.0613 \end{array} \right) \right)
\]

QUADPOWER VARIATION

\[
\frac{1}{\delta^{1/2} \sqrt{\int_0^t \sigma_u^4 du}} \left( \mu_1^{-4} \sum_{j=1}^{t/\delta-3} \sqrt{y_{j,i}^2 y_{j+1,i}^2 y_{j+2,i}^2 y_{j+3,i}^2} - \int_0^t \sigma_u^2 du \right) \rightarrow N \left( 0, \left( \begin{array}{cc} 2 & 2 \\ 2 & 3.3704 \end{array} \right) \right)
\]
TESTS FOR JUMPS

$H_0$: The price process does not include a jump component.

$H_1$: The price process consists of a continuous and jump component.

Assume the jump component is a finite activity jump process.

**Linear Test-Statistics**

\[
\frac{1}{\delta^{1/2} \sqrt{\int_0^t \sigma_s^4 ds}} \left( \mu_{2/3}^{-3} \{ Y_{\delta} \}^{[2/3,2/3,2/3]}_t - [Y_{\delta}]^{[2]}_t \right) \xrightarrow{L} N(0, \vartheta_{TV})
\]

where $\vartheta_{TV} = \mu_{4/3} \mu_{2/3}^{-2} (\mu_{4/3}^2 \mu_{2/3}^{-4} + 2 \mu_{4/3} \mu_{2/3}^{-2} - 2) - 7 \approx 1.0613$

\[
\frac{1}{\delta^{1/2} \sqrt{\int_0^t \sigma_s^4 ds}} \left( \mu_{1/2}^{-4} \{ Y_{\delta} \}^{[1/2,1/2,1/2,1/2]}_t - [Y_{\delta}]^{[2]}_t \right) \xrightarrow{L} N(0, \vartheta_{QV})
\]

where $\vartheta_{QV} = \mu_1 \mu_{1/2}^{-2} (\mu_1^3 \mu_{1/2}^{-6} + 2 \mu_1^2 \mu_{1/2}^{-4} + 2 \mu_1 \mu_{1/2}^{-2} - 2) - 9 \approx 1.37702$

\[
\frac{1}{\delta^{1/2} \sqrt{\int_0^t \sigma_s^4 ds}} \left( \mu_1^{-2} \{ Y_{\delta} \}^{[1,0,1]}_t - [Y_{\delta}]^{[2]}_t \right) \xrightarrow{L} N(0, \vartheta_{SBV})
\]

where $\vartheta_{SBV} = \mu_1^{-4} + 2 \mu_1^{-2} - 5 \approx 0.60907$
Ratio Test-Statistics

\[
\frac{\left( \frac{\mu_{2/3} \{ Y_\delta \}^{[2/3, 2/3, 2/3]}_t - 1}{[Y_\delta]_t^{[2]}} \right)}{\delta^{1/2} \sqrt{\frac{\int_0^t \sigma_s^4 ds}{\left( \int_0^t \sigma_s^2 ds \right)^2}}} \quad \xrightarrow{L} \quad N(0, \vartheta_{TV})
\]

where \( \vartheta_{TV} \simeq 1.0613; \)

\[
\frac{\left( \frac{\mu_{1/2} \{ Y_\delta \}^{[1/2, 1/2, 1/2, 1/2]}_t - 1}{[Y_\delta]_t^{[2]}} \right)}{\delta^{1/2} \sqrt{\frac{\int_0^t \sigma_s^4 ds}{\left( \int_0^t \sigma_s^2 ds \right)^2}}} \quad \xrightarrow{L} \quad N(0, \vartheta_{QV})
\]

where \( \vartheta_{QV} \simeq 1.37702; \)

\[
\frac{\left( \frac{\mu_{1} \{ Y_\delta \}^{[1, 0, 1]}_t - 1}{[Y_\delta]_t^{[2]}} \right)}{\delta^{1/2} \sqrt{\frac{\int_0^t \sigma_s^4 ds}{\left( \int_0^t \sigma_s^2 ds \right)^2}}} \quad \xrightarrow{L} \quad N(0, \vartheta_{SBV})
\]

where \( \vartheta_{SBV} \simeq 0.60907. \)
Integrated Quarticity

1) Realised Quarticity (E1)

\[ M \mu_4^{-1} \{ Y_M \}_{i}^{[4]} = M \mu_4^{-1} \sum_{j=1}^{M} y_{j,i}^4 \]

2) Realised Tripower Variation with \( r = s = u = 4/3 \) (E2)

\[ M \mu_{4/3}^{-3} \{ Y_M \}_{i}^{[4/3,4/3,4/3]} = M \mu_{4/3}^{-3} \sum_{j=1}^{M-2} | y_{j,i}^{4/3} | y_{j+1,i}^{4/3} | y_{j+2,i}^{4/3} \]

3) Realised Quadpower Variation with \( r = s = u = v = 1 \) (E3)

\[ M \mu_1^{-4} \{ Y_M \}_{i}^{[1,1,1,1]} = M \mu_1^{-4} \sum_{j=1}^{M-3} | y_{j,i} || y_{j+1,i} || y_{j+2,i} || y_{j+3,i} |. \]

One-sided test

\[ \text{min} \left( 0, \mu_{2/3}^{-3} \{ Y_M \}_{i}^{[2/3,2/3,2/3]} - [Y_M]^{[2]} \right) \]

Modified estimators

\[ \left( \frac{M}{M-2} \right) \mu_{2/3}^{-3} \{ Y_M \}_{i}^{[2/3,2/3,2/3]}, \quad \left( \frac{M}{M-3} \right) \mu_{1/2}^{-4} \{ Y_M \}_{i}^{[1/2,1/2,1/2,1/2]}, \quad \left( \frac{M}{M-2} \right) \mu_1^{-2} \{ Y_M \}_{i}^{[1,0,1]} \]
Integrated Quarticity

1) Realised Quarticity (E1)

\[ M_{\mu_4^{-1}} \{ Y_M \}_i^{[4]} = M_{\mu_4^{-1}} \sum_{j=1}^{M} y_{j,i}^4 \]

2) Realised Tripower Variation with \( r = s = u = 4/3 \) (E2)

\[ M_{\mu_{4/3}^{-3}} \{ Y_M \}_i^{[4/3,4/3,4/3]} = M_{\mu_{4/3}^{-3}} \sum_{j=1}^{M-2} y_{j,i}^{4/3} y_{j+1,i}^{4/3} y_{j+2,i}^{4/3} \]

3) Realised Quadpower Variation with \( r = s = u = v = 1 \) (E3)

\[ M_{\mu_1^{-4}} \{ Y_M \}_i^{[1,1,1,1]} = M_{\mu_1^{-4}} \sum_{j=1}^{M-3} y_{j,i} y_{j+1,i} y_{j+2,i} y_{j+3,i} \]

One-sided test

\[ \min \left( 0, \mu_{2/3}^{-3} \{ Y_M \}_i^{[2/3,2/3,2/3]} - [Y_M]_i^{[2]} \right) \]

Modified estimators

\[ \left( \frac{M}{M-3} \right)^{\mu_{1/2}^{-4}} \{ Y_M \}_i^{[1/2,1/2,1/2,1/2]} \]

\[ \left( \frac{M}{M-2} \right)^{\mu_1^{-2}} \{ Y_M \}_i^{[1,0,1]} \]
Integrated Quarticity

1) Realised Quarticity (E1)

\[ M_{\mu_0^{-1}} \{ Y_M \}_i^{[4]} = M_{\mu_0^{-1}} \sum_{j=1}^{M} y_{j,i}^4 \]

2) Realised Tripower Variation with \( r = s = u = \frac{4}{3} \) (E2)

\[ M_{\mu_{4/3}^{-3}} \{ Y_M \}_i^{[4/3,4/3,4/3]} = M_{\mu_{4/3}^{-3}} \sum_{j=1}^{M-2} | y_{j,i} |^{4/3} | y_{j+1,i} |^{4/3} | y_{j+2,i} |^{4/3} \]

3) Realised Quadpower Variation with \( r = s = u = v = 1 \) (E3)

\[ M_{\mu_1^{-4}} \{ Y_M \}_i^{[1,1,1,1]} = M_{\mu_1^{-4}} \sum_{j=1}^{M-3} | y_{j,i} | | y_{j+1,i} | | y_{j+2,i} | | y_{j+3,i} | \]

One-sided test

\[ \min \left( 0, \mu_{2/3}^{-3} \{ Y_M \}_i^{[2/3,2/3,2/3]} - [Y_M]_i^{[2]} \right) \]

Modified estimators

\( \left( \frac{M}{M-2} \right)^{\mu_{2/3}^{-3}} \{ Y_M \}_i^{[2/3,2/3,2/3]} \), \( \left( \frac{M}{M-3} \right)^{\mu_{1/2}^{-4}} \{ Y_M \}_i^{[1/2,1/2,1/2,1/2]} \), \( \left( \frac{M}{M-2} \right)^{\mu_1^{-2}} \{ Y_M \}_i^{[1,0,1]} \).
Simulations

Square Root Process (Cox, Ingersoll and Ross (1985))

\[ d\sigma_t^2 = -\lambda (\sigma_t^2 - \xi) dt + \omega \sigma_t dB_{\lambda t}, \quad \xi \geq \omega^2/2, \quad \lambda > 0, \]

where \( B \) is a standard Brownian motion process.

The square root process has a marginal distribution

\[ \sigma_t^2 \sim \Gamma(2\omega^{-2}\xi, 2\omega^{-2}) = \Gamma(\nu, a), \quad \nu \geq 1, \]

with a mean of \( \xi = \nu/a \) and a variance of \( \omega^2 = \nu/a^2 \).

We will take \( A_t = 0 \) and rule out the leverage effect by assuming \( \text{Cor}\{B_{\lambda t}, W_t\} = 0 \). We will take \( h = 1, \lambda = 0.01, \nu = 4 \) and \( a = 8 \).

The jumps will be i.i.d. \( N(0, \beta \nu/a) \), thus a jump has the same variance as that expected over a \((\beta \times 100)\)% of a day when there are no jumps.
### Infeasible Tests

<table>
<thead>
<tr>
<th>M</th>
<th>Bias</th>
<th>12 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>72 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>288 SD</th>
<th>Cove</th>
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</thead>
<tbody>
<tr>
<td>RBV</td>
<td>−.363</td>
<td>1.16</td>
<td>87.4</td>
<td>−.191</td>
<td>1.00</td>
<td>92.8</td>
<td>−.115</td>
<td>1.00</td>
<td>93.5</td>
</tr>
<tr>
<td>RTV</td>
<td>−.313</td>
<td>1.13</td>
<td>88.5</td>
<td>−.180</td>
<td>1.00</td>
<td>92.9</td>
<td>−.103</td>
<td>.994</td>
<td>94.1</td>
</tr>
<tr>
<td>RQV</td>
<td>−.313</td>
<td>1.14</td>
<td>89.0</td>
<td>−.181</td>
<td>1.00</td>
<td>93.3</td>
<td>−.104</td>
<td>.999</td>
<td>94.3</td>
</tr>
<tr>
<td>RSBV</td>
<td>−.424</td>
<td>1.22</td>
<td>85.9</td>
<td>−.199</td>
<td>1.03</td>
<td>92.2</td>
<td>−.129</td>
<td>1.01</td>
<td>93.6</td>
</tr>
</tbody>
</table>

*Bias, standard deviation and coverage (95% level) of the infeasible linear test.*

### Feasible Tests

<table>
<thead>
<tr>
<th>M</th>
<th>Bias</th>
<th>E1 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>E2 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>E3 SD</th>
<th>Cove</th>
</tr>
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<tbody>
<tr>
<td>RBV</td>
<td>−.086</td>
<td>.997</td>
<td>93.5</td>
<td>−.165</td>
<td>1.03</td>
<td>91.6</td>
<td>−.173</td>
<td>1.04</td>
<td>91.3</td>
</tr>
<tr>
<td>RTV</td>
<td>−.075</td>
<td>.992</td>
<td>94.3</td>
<td>−.154</td>
<td>1.02</td>
<td>92.1</td>
<td>−.167</td>
<td>1.03</td>
<td>91.6</td>
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<tr>
<td>RQV</td>
<td>−.076</td>
<td>.999</td>
<td>94.8</td>
<td>−.149</td>
<td>1.02</td>
<td>92.8</td>
<td>−.167</td>
<td>1.04</td>
<td>91.7</td>
</tr>
<tr>
<td>RSBV</td>
<td>−.103</td>
<td>1.01</td>
<td>94.4</td>
<td>−.161</td>
<td>1.04</td>
<td>92.9</td>
<td>−.175</td>
<td>1.05</td>
<td>92.2</td>
</tr>
</tbody>
</table>

*Bias, standard deviation and coverage (95% level) of the feasible linear test for M=288.*

### Feasible Tests

<table>
<thead>
<tr>
<th>M</th>
<th>Bias</th>
<th>E1 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>E2 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>E3 SD</th>
<th>Cove</th>
</tr>
</thead>
<tbody>
<tr>
<td>RBV</td>
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<td>94.9</td>
<td>−.110</td>
<td>.980</td>
<td>93.7</td>
<td>−.113</td>
<td>.980</td>
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<td>.976</td>
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<td>.998</td>
<td>97.1</td>
<td>−.087</td>
<td>.968</td>
<td>94.8</td>
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<td>.980</td>
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</tr>
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<td>.994</td>
<td>95.0</td>
<td>−.117</td>
<td>.985</td>
<td>94.1</td>
<td>−.124</td>
<td>.989</td>
<td>94.0</td>
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</tbody>
</table>

*Bias, standard deviation and coverage (95% level) of the feasible ratio test with M=288.*
### Infeasible

<table>
<thead>
<tr>
<th>M</th>
<th>Bias</th>
<th>12 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>72 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>288 SD</th>
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<tr>
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<td>69.8</td>
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<td>6.42</td>
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<td>89.9</td>
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<td>-.399</td>
<td>1.46</td>
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</tbody>
</table>

Bias, standard deviation and coverage (95% level) of the infeasible linear test.

### Feasible

<table>
<thead>
<tr>
<th>M</th>
<th>Bias</th>
<th>E1 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>E2 SD</th>
<th>Cove</th>
<th>Bias</th>
<th>E3 SD</th>
<th>Cove</th>
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<td>.782</td>
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<td>-.154</td>
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<td>62.4</td>
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<td>88.4</td>
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</table>

Bias, standard deviation and coverage (95% level) of the feasible ratio test when one jump is added every day for M=288.
### SIZE ADJUSTED TESTS

#### Infeasible

<table>
<thead>
<tr>
<th>M</th>
<th>β</th>
<th>CV</th>
<th>12%</th>
<th>Pow 50%</th>
<th>Pow 20%</th>
<th>CV</th>
<th>72%</th>
<th>Pow 50%</th>
<th>Pow 20%</th>
<th>CV</th>
<th>288%</th>
<th>Pow 50%</th>
<th>Pow 20%</th>
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<tbody>
<tr>
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</tr>
</tbody>
</table>

Critical values (5% size), Normal quantiles and power of the infeasible linear tests when one jump is added every day.

#### Feasible

<table>
<thead>
<tr>
<th>β</th>
<th>CV</th>
<th>E1%</th>
<th>Pow 50%</th>
<th>Pow 20%</th>
<th>CV</th>
<th>E2%</th>
<th>Pow 50%</th>
<th>Pow 20%</th>
<th>CV</th>
<th>E3%</th>
<th>Pow 50%</th>
<th>Pow 20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>BV</td>
<td>1.63</td>
<td>94.9</td>
<td>0.08</td>
<td>0.05</td>
<td>1.79</td>
<td>96.4</td>
<td>0.33</td>
<td>0.08</td>
<td>1.80</td>
<td>96.5</td>
<td>0.33</td>
<td>0.08</td>
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<td>TV</td>
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<td>0.06</td>
<td>1.75</td>
<td>96.1</td>
<td>0.32</td>
<td>0.06</td>
</tr>
<tr>
<td>QV</td>
<td>1.46</td>
<td>92.8</td>
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<td>0.05</td>
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<td>1.70</td>
<td>95.6</td>
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<tr>
<td>SBV</td>
<td>1.63</td>
<td>94.9</td>
<td>0.09</td>
<td>0.05</td>
<td>1.78</td>
<td>96.3</td>
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<td>0.07</td>
<td>1.77</td>
<td>96.2</td>
<td>0.34</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Critical values, Normal quantiles and power of the feasible ratio tests for M=288 when one jump is added every day.
Empirical Data

Dollar/Deutsche Mark exchange rates. (Olsen and Associates)

From 1st of December 1986 until 30th of November 1996

Quotes from Reuters screen

Five-minutes data (Market Microstructure Effect)
Sum of the first five correlation coefficients of the Dollar/DM series (corr). Average value of realised variance (RV), realised bipower (BV), tripower (TV), quadpower (QV) and skipped bipower (SB) variation. Proportion of rejections of the null hypothesis at the 5% and 1% level.
CONCLUSIONS

Multipower variation provides a tool to test for the presence of jumps in log-price process.

Tests based on Tripower and Quadpower variation seem to have better size.

Tripower or Quadpower variation give better estimations of integrated quarticity.

Study the effect of Market Microstructure Noise.